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A SOCIAL PRAGMATIC VIEW ON THE CONCEPT OF NORMATIVE CONSISTENCY

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ABSTRACT

The programmatic statement put forward in von Wright's last works on deontic logic introduces the perspective of logical pragmatics, which has been formally explicated here and extended so to include the role of norm-recipient as well as the role of norm-giver. Using the translation function from the language of deontic logic to the language of set-theoretical approach, the connection has been established between the deontic postulates, on one side, and the perfection properties of the norm-set and the counter-set, on the other side. In the study of conditions of rational norm-related activities it has been shown that diverse dynamic second-order norms related to the concept of the consistency norm-system hold: – the norm-giver ought to restore "classical" consistency by revising an inconsistent system, – the norm-recipient ought to preserve an inconsistent system by revision of its logic so that inconsistency does not imply destruction of the system. Dialetheic deontic logic of Priest is a suitable logic for the purpose since it preserves other perfection properties of the system.

Keywords: concepts of normativity, deontic logic, logical pragmatics

In one of his later works on deontic logic von Wright put forward a programmatic statement on its nature:

> Deontic logic, one could also say, is neither a logic of norms nor a logic of norm-propositions but *a study of conditions which must be satisfied in rational norm-giving activity*. (von Wright 1993, 111)

In this paper an outline of a formal system will be proposed as a possible explication for the new perspective in deontic logic envisaged by von Wright. The basic idea will be to connect by translation the axioms of standard deontic logic with descriptions of "perfection properties" that a normative system should have and to formulate corresponding second-order obligations, or requirements of rationality to which the norm-giver is subordinated in the norm-giving activity and the norm-recipient in the normative reasoning, including the corrective obligation of logic revision.

1. Filters and ideals

In the discussion that will follow it will be assumed that actors relate to the logical structures of the concepts of obligation and permission. Therefore the two logical structures, the filter and the weak ideal structure, will be introduced since they will be used in the exposition.

The set-theoretic structure that corresponds to a consistent theory or consistent deductively closed set is called a "filter". A filter F is defined as a set of subsets of a given set W satisfying the following conditions (Jech 2003, 73):

1. $\emptyset \notin F$,
2. $W \in F$,
3. if $X \in F$ and $Y \in F$, then $X \cap Y \in F$,
4. for all $X, Y \subseteq W$, if $X \in F$ and $X \subseteq Y$, then $Y \in F$.

In the case of classical propositional logic the correspondence between the sentences of a deductively closed theory, $T = Cn(T)$, and a filter is achieved by collecting all the truth-sets (the sets of verifying valuations) of sentences in the theory. The set of sets of valuations $\{[\![\varphi]\!] \mid \varphi \in Cn(T)\}$ is a filter if $Cn(T)$ is consistent, where $[\![\varphi]\!] = \{w \mid w(\varphi) = \mathrm{t}\}$ is the set of valuations w verifying φ. The filter conditions can be reformulated in syntactic terms. To the condition 3. there corresponds closure under conjunction since $[\![\varphi \wedge \psi]\!] = [\![\varphi]\!] \cap [\![\psi]\!]$. To the relation of entailment from φ to ψ there corresponds an inclusion relation $[\![\varphi]\!] \subseteq [\![\psi]\!]$. So, the syntactic condition corresponding to condition 4. is that if $\varphi \in Cn(T)$ and φ entails ψ, then $\psi \in Cn(T)$.

The intersection of all truth-sets of sentences of T, i.e., $\bigcap \{[\![\varphi]\!] \mid \varphi \in T\}$, is the semantic base of the theory $Cn(T)$ and any truth-set contained in the filter includes it. If the semantic base is a singleton set containing exactly one valuation w, $\{w\} = \bigcap \{[\![\varphi]\!] \mid \varphi \in T\}$, then for any set X of valuations it holds that either the set X or its complement $W - X$ contains w. Thanks to this fact the filter is "exhaustive": it contains exactly one member from the pair of a set and its complement. A filter F satisfying this condition (i.e., for any $X \subseteq W$ either $X \in F$ or $(W - X) \in F$) is called 'maximal filter' or 'ultrafilter'. Let us note in passing that in Gödel's theory of properties the set of positive properties has the structure of a maximal filter.

The set-theoretic structure corresponding to the "counter-theory", $\mathscr{L} - Cn(T)$ (where $\mathscr{L}$ is the language of T), is closed in the opposite direction: if $[\![\varphi]\!]$ corresponds to some $\varphi \in (\mathscr{L} - Cn(T))$, then so does any subset of it, i.e., if $\varphi \in (\mathscr{L} - Cn(T))$ and $[\![\psi]\!] \subseteq [\![\varphi]\!]$, then $\psi \in (\mathscr{L} - Cn(T))$. The theory T either excludes some information or leaves it indeterminate, so, any truth-set contained in the excluded or indeterminate information must belong to the set-theoretic structure corresponding to the counter-theory. The special subtype of this set-theoretical structure is called the 'ideal'. An ideal I is defined as a set of subsets of a given set W satisfying the following conditions (Jech 2003, 73):

1. $\emptyset \in I$,
2. $W \notin I$,
3. if $X \in I$ and $Y \in I$, then $X \cup Y \in I$,
4. for all $X, Y \subseteq W$, if $X \in I$ and $Y \subseteq X$, then $Y \in I$.

The conditions can be reformulated in syntactic terms. To condition 3. there corresponds closure under disjunction if both disjuncts are already in the set. To the relation of entailment from φ to ψ there corresponds inclusion relation $[\![\varphi]\!] \subseteq [\![\psi]\!]$. So, the syntactic condition corresponding to condition 4. is that if $\psi \in (\mathscr{L} - Cn(T))$ and φ entails ψ, then $\varphi \in (\mathscr{L} - Cn(T))$.

Contrary to the claim put forward in (Jech 2003, 73), it does not generally hold that an ideal is the complement of a filter, and it does not generally hold that a filter is the complement of an ideal. Let us consider a counterexample to see that the ideal–filter duality does not hold in general! Let $\mathscr{L}$ be a language of propositional logic built over two propositional letters p and q and let $W = \{w_{pq}, w_p, w_q, w_\emptyset\}$ be the set of valuations where the presence of a letter in the subscript indicates that a valuation assigns the value 'true' to the letter. Suppose $T = \{p\}$. Then the filter corresponding to $Cn(T)$ is the set $F = \{\{w_{pq}, w_p\}, \{w_{pq}, w_p, w_q\}, \{w_{pq}, w_p, w_\emptyset\}, W\}$. Since the theory $Cn(T)$ is not complete, the counter-theory $\mathscr{L} - Cn(T)$ has both q and $\neg q$. If the set-theoretic structure corresponding to $\mathscr{L} - Cn(T)$ is an ideal, then it will contain $[\![q]\!] \cup [\![\neg q]\!] = W$, but this is not allowed by the definition of the ideal. Therefore the complement of a filter need not always be an ideal. On the other hand, if a theory is complete, its corresponding filter is maximal, and its complement is an ideal.[1]

Although the complement of a filter needs not be an ideal, it shares an essential property of the ideal, the property of "closure under implicants" as the first item in Proposition 1.1 shows.

[1]Let $I = W - F$ and let F be a maximal filter. To prove condition 3, suppose that $[\![\varphi]\!] \in I$ and $[\![\psi]\!] \in I$. So, $W - [\![\varphi]\!] \in F$ and $W - [\![\psi]\!] \in F$ and, by the set-theory, (*) $W - ([\![\varphi]\!] \cup [\![\psi]\!]) \in F$. For reductio suppose that $[\![\varphi]\!] \cup [\![\psi]\!] \notin I$. So, $[\![\varphi]\!] \cup [\![\psi]\!] \in F$. This is impossible by (*) since F has a non-empty intersection if it is maximal. To prove condition 4. assume that $[\![\varphi]\!] \subseteq [\![\psi]\!]$ and (**) $[\![\psi]\!] \in I$. For reductio suppose that $[\![\varphi]\!] \notin I$. So, $[\![\varphi]\!] \in F$. Then, by the definition of filter, $[\![\psi]\!] \in F$, which is impossible because of (**).

Proposition 1.1. *If $S = W - F$ and F is a filter, then*

1. *if* $[\![\varphi]\!] \subseteq [\![\psi]\!]$ *and* $[\![\psi]\!] \in S$, *then* $[\![\varphi]\!] \in S$,
2. *if* $[\![\varphi]\!] \cap [\![\psi]\!] \in S$, *then* $[\![\varphi]\!] \in S$ *or* $[\![\psi]\!] \in S$,

Proof. For 1. use reductio. Suppose $[\![\varphi]\!] \notin S$. Then $[\![\varphi]\!] \in F$. Since F is closed under superset relation, $[\![\psi]\!] \in F$. This is impossible since $[\![\psi]\!] \in S$. For 2. use reductio. Suppose $[\![\varphi]\!] \notin S$ and $[\![\psi]\!] \notin S$. Then $[\![\varphi]\!] \in F$ and $[\![\psi]\!] \in F$. Since F is a filter it follows that $[\![\varphi]\!] \cap [\![\psi]\!] \in F$. This is impossible since $[\![\varphi]\!] \cap [\![\psi]\!] \in S$. □

Definition 1.1 (Weak ideal)**.** A structure S is a weak ideal iff (i) $[\![\varphi]\!] \subseteq [\![\psi]\!]$ and $[\![\psi]\!] \in S$, then $[\![\varphi]\!] \in S$, and (ii) if $[\![\varphi]\!] \cap [\![\psi]\!] \in S$, then $[\![\varphi]\!] \in S$ or $[\![\psi]\!] \in S$.

2. Normative system as a set of sentences

The concept of normativity has received different explications in contemporary philosophy. Among these the modal approach and set-theoretic approach stand out.[2] In the set theoretic approach the basic idea is to represent the normative force by the membership relation between a propositional content φ and a certain set $\mathcal{N}$, "norm-set". The expression 'it is obligatory that φ' is explicated as '$\varphi \in \mathcal{N}$'. This highly reduced model can be made more realistic by adding more variables, such as those for the source, addressee and situation (Broome 2013, 117) and taking 'by the source s it is obligatory in the situation w upon actor i that φ' as the elementary sentence; the elementary sentence is translated as '$\varphi \in \mathcal{N}(s, w, i)$' where the set of required propositions is the value of the three-place function $\mathcal{N}(s, w, i)$. Nevertheless the basic idea remains the same. The major point of divergence within the set-theoretic approach lies in the properties one is willing to assign to norm-sets. For example, Alchourrón and Bulygin (1998) treat obligation norm-sets as deductively closed sets.[3] On the other hand, Broome (2013, 122) only requires norm-sets to be closed under equivalence. Nevertheless, there are no obstacles in treating, as will be done here, the norm-sets as simple sets consisting just of sentences that correspond to contents of explicitly promulgated norms and

[2] Standard deontic logic is the KD normal modal logic having axioms (K) $O(\varphi \to \psi) \to (O\varphi \to O\psi)$ and (D) $O\varphi \to \neg O \neg \varphi$, and the necessitation rule $\vdash \varphi \Rightarrow \vdash O\varphi$. For the purposes of this text the axiomatization given in (Kanger and Kanger 2001) with the rule of inheritance along entailment relation (if φ entails ψ, then $O\varphi \to O\psi$), the adjunction axiom (2) and axiom (D) is very convenient.

[3] Alchourrón and Bulygin (1998, 391): "We (...) define the concept of a normative system as the set of all the propositions that are consequences of the explicitly commanded propositions. (...) This enables us to distinguish between the set A (formed by all the explicitly commanded propositions) as the axiomatic basis of the system, and the normative system $Cn(A)$, which is the set of all the consequences of A.".

to lay the question of their logical properties aside. Furthermore, it is in accord with the approach proposed by von Wright to treat norm-sets as simple sets.

The set theoretic and modal approach need not be viewed as alternative perspectives on the same phenomenon but rather as the two languages used in its description. As suggested in (von Wright 1999) some deontic axioms describe the desirable properties of a norm-set and exactly these properties the norm-giver ought to achieve in a rational norm-giving activity. In (Žarnić 2010) the translation function has been introduced for translating deontic formulas from the language without iterated modalities to the set theoretic language and in (Žarnić and Bašić 2014) the extension of the pragmatic reading of deontic axioms has been proposed so to include the roles of the norm-subject and the norm-applier.

The reasons why the term 'social pragmatics' is used are, inter alia, the following. The term 'pragmatics' indicates the study of language-use. The deontic logic conceived along the lines of a generalized version of von Wright's programmatic statement (as the study of rationality conditions of *norm-related* activities) includes the study of the use of language in the building of a normative system and in normative reasoning. The term 'social' indicates that more than one language-user (or social role) is taken into account, and here the logical structure of normativity will be investigated from the perspectives of the two roles, the norm-giver and the norm-recipient role. Thus, social pragmatics of deontic logic studies the norms that apply to norm-related activities of social actor roles. These norms can be properly called 'second-order norms' since they cover the activities that are related to a normative-system.

Definition 2.1. Language $\mathscr{L}_{\mathrm{O}}$ is a deontic language without iterated modalities: $\varphi ::= p \mid \mathrm{O}\varphi \mid \mathrm{P}\varphi \mid \neg\varphi \mid (\varphi_1 \vee \varphi_2)$, where p is a sentence of language $\mathscr{L}_{pl}$ of propositional logic. The definitions of deontic modality F and of truth-functional connectives are standard. Language $\mathscr{L}_{\mathscr{N}}$ is the language of the norm-set membership of the set-theoretic approach: $\varphi ::= p \mid \ulcorner p \urcorner \in \mathscr{N} \mid \neg\varphi \mid (\varphi_1 \vee \varphi_2)$, where $p \in \mathscr{L}_{pl}$.[4]

Definition 2.2. Function $\tau^+ : \mathscr{L}_{\mathrm{O}} \mapsto \mathscr{L}_{\mathscr{N}}$ translates formulas of the deontic language without iterated modalities to the language of the norm-set membership.

$$\begin{aligned}
&\tau^+(\varphi) = \varphi \qquad \text{if } \varphi \in \mathscr{L}_{pl}\\
&\tau^+(\mathrm{O}\varphi) = \ulcorner\varphi\urcorner \in \mathscr{N}\\
&\tau^+(\mathrm{P}\varphi) = \ulcorner\neg\varphi\urcorner \notin \mathscr{N}\\
&\tau^+(\mathrm{P}\neg\varphi) = \ulcorner\varphi\urcorner \notin \mathscr{N}\\
&\tau^+(\neg\varphi) = \neg\tau^+(\varphi)\\
&\tau^+((\varphi \vee \psi)) = (\tau^+(\varphi) \vee \tau^+(\psi))
\end{aligned}$$

[4] "Quine quotes", $\ulcorner\varphi\urcorner$, indicate that the expression φ is mentioned, not used. So, $\ulcorner\varphi\urcorner$ names φ.

Consistency or seriality axiom (D), $O\varphi \to \neg O \neg\varphi$, characterizes *consistent* norm-sets as shown by the translation: $\tau^+(O\varphi \to \neg O\neg\varphi) = \ulcorner\varphi\urcorner \in \mathscr{N} \to \ulcorner\neg\varphi\urcorner \notin \mathscr{N}$. Adjunction axiom (2), $(O\varphi \wedge O\psi) \to O(\varphi \wedge \psi)$ characterizes norm-sets *closed under conjunction* as shown by the translation: $\tau^+((O\varphi \wedge O\psi) \to O(\varphi \wedge \psi)) = (\ulcorner\varphi\urcorner \in \mathscr{N} \wedge \ulcorner\psi\urcorner \in \mathscr{N}) \to \ulcorner\varphi \wedge \psi\urcorner \in \mathscr{N}$. It may be thought that both consistency and closure under conjunction are desirable properties, i.e., the properties that any norm-set ought to have, but their desirability is relative to the actor's role. From the perspective of the norm-giver consistency is a property that ought to be achieved in the norm-giving activity, but closure under conjunction is not since it would involve an infinite sequence of communicative acts of adding still another conjunction for any pair of sentences already achieved in the process. From the perspective of the norm-recipient consistency of a norm-set is not a property that the norm-recipient ought to achieve in her/his normative reasoning since she/he ought to reason on the basis of the norm-set no matter whether it is consistent or not. On the other hand, closure under conjunction is the property related to the norm-recipient's reasoning since she/he ought to arrive at the minimal (non-redundant) conjunctive conclusion of her/his obligations.

Axiom (K), $O(\varphi \to \psi) \to (O\varphi \to O\psi)$, in the presence of the necessitation rule characterizes *deductively closed* norm-sets as shown by the translation: $\tau^+(O(\varphi \to \psi) \to (O\varphi \to O\psi)) = \ulcorner\varphi \to \psi\urcorner \in \mathscr{N} \to (\ulcorner\varphi\urcorner \in \mathscr{N} \to \ulcorner\psi\urcorner \in \mathscr{N})$. Alternatively but in the same direction, the related property of *closure under entailment* is characterized by the rule of deontic inheritance *if* φ *entails* ψ, *then* $O\varphi \to O\psi$; this could be a more convenient solution since it does not require that the norm-set has all the logical truths. Extending the line of thought to the Roman Law principle (*ultra posse nemo obligatur*, *impossibilium nulla obligatio*) and introducing the operator Can_r for "doability" or the recipient's r ability to see to it that so-and-so is the case, $O\varphi \to \mathsf{Can}_r\varphi$, it is clear that the principle characterizes *doable* norm-sets as shown by the translation: $\tau^+(O\varphi \to \mathsf{Can}_r\varphi) = \ulcorner\varphi\urcorner \in \mathscr{N} \to \mathsf{Can}_r\varphi$. It is obligatory for the norm-recipient to draw deductive consequences of explicitly stated norms and not obligatory for the norm-giver to proclaim them. So, only from the norm-recipient's perspective the deductive closure of $\mathscr{N}$ is its perfection property, i.e., in normative reasoning she or he ought to relate to the deductively closed set, and not only to its deductive core. On the other hand, the Roman Law principle seems to allow for both interpretations: it is clear that the norm-giver ought to achieve a "doable" norm-set, but is disputable whether the norm-recipient is licensed to conclude that a norm is not binding if its content is not doable or under obligation to revise her/his theory of doability.

From the point of view of social pragmatics the identity of the actor's role (with respect to the norm-set) must be taken into account in the analysis of a logical phenomenon. It can be said that the roles relate to the same concept of obligation, conceived as the membership of a sentence in the norm-set, but their perspectives differ and thus reveal different properties of the norm-set. From the perspective of the norm-giver role the consistency of the norm-set appears as s normative property, the property that ought to be achieved in the norm-giving activity. On the other hand, from the perspective of the norm-recipient consistency is a descriptive property that the norm-set either has or has not. Similarly, but reversely, the property of closure under entailment is a normative property from the norm-recipient's perspective since any deductive consequence of the norm-set ought to be accepted in the norm-recipient's normative reasoning. As noted in the example from (Goble 2009), no defence for camping on a street on Thursday can be built on the ground that there is only a norm prohibiting camping on a street on any day of week but no explicit norm regarding Thursday. The reason being that the norm-recipient in his normative reasoning ought to relate to the norm-set as being deductively closed. The norm-giving activity does not relate to the closure property since there can be no empirical object corresponding to the deductively closed norm-set as it is necessarily infinite.

Suppose that one wants to add $\mathrm{P}\varphi \rightarrow \mathrm{O}\varphi$ as an axiom and thus exclude optionality. The axiom characterizes frames where accessibility relation is functional, $\forall x \forall y \forall z((Rxy \wedge Rxz) \rightarrow y = z)$. The translation $\tau^{+}(\mathrm{P}\varphi \rightarrow \mathrm{O}\varphi) = \ulcorner \neg\varphi \urcorner \notin \mathscr{N} \rightarrow \ulcorner \varphi \urcorner \in \mathscr{N}$ shows that the axiom characterizes complete norm-sets. If the language of $\mathscr{N}$ is rich enough to express its own syntax then a finite $\mathscr{N}$ cannot be complete. So, in this case completeness cannot be a desirable property of a finite norm-set if one accepts the third order norm that a property is desirable only if it is logically possible. If so, then the semantic structure corresponding to deductively closed set cannot be an ultrafilter.

3. Non-conceptual connections between obligations and permissions

In this section an extension of the set-theoretic approach will be proposed in order to capture von Wright's non-derivative concept of permission, i.e., permission and obligation are not interdefinable. So, besides the norm-set created by explicit obligation-norms, another set is required in the model, namely, the one which is built by permission-norms.

> Just as possibility is the negation of the necessity of the contradictory of a proposition, permission is the negation of the obligatoriness of the contradictory. $\mathrm{P}p \leftrightarrow \neg\mathrm{O}\neg p$ is a theorem of "classical" deontic logic.
>
> I think that this opinion is mistaken. The relation between permission and absence of prohibition is not a *conceptual* but a *normative* relation. One may be able to give good reasons why such things which are not

> prohibited by the norms of a certain code should be regarded as permitted by the code in question. But to declare the non-prohibited permitted is a normative act. One could have a meta-norm to the effect that the not-prohibited is permitted. The well-known principles *Nulla poena sine lege* and *Nullum crimen sine lege* may be thought of as versions of this meta-norm. Or at least as closely related to it. (von Wright 1991, 179)

If permission is not the absence of prohibition, what is it? Several answers have been discussed in the literature. An incomplete list follows with an additional proposal at the end:

- To give a permission is to remove an antecedently existing prohibition.
- Giving permission to an actor to see to it that something is the case implies prohibiting any other actor to prevent her/him from doing so.
- To give a permission means to declare that something is optional. To give a permission for φ means to simultaneously introduce two permission-norms: $\mathrm{P}\varphi$ and $\mathrm{P}\neg\varphi$.[5]

Here the disputes on the meaning/s of 'permission' will be set aside; so it will be taken in its weakest non-derivative sense.

In order to capture the concept of permission in a non-derivative sense, the "counter-set", $\overline{\mathscr{N}}$, will be introduced in the model. Within the two-sets model the scope of "perfection properties" becomes wider. Besides perfection properties of individual sets, there are also *perfection properties of the relation* between them. Gaplessness is one of these.

Let $\mathscr{L}$ be the language in which the norm-system is formulated. Norm-system $\langle \mathscr{N}, \overline{\mathscr{N}} \rangle$ is gapless iff $\mathscr{N} \cup \overline{\mathscr{N}} = \mathscr{L}$. The operation corresponding to the introduction of a permission-norm is the addition of the contradiction of its content to the counter-set: $\ulcorner \neg\varphi \urcorner$ is added to $\overline{\mathscr{N}}$ if $\mathrm{P}\varphi$, $\ulcorner \varphi \urcorner$ is added to $\overline{\mathscr{N}}$ if $\mathrm{P}\neg\varphi$. Considering the case of $\mathrm{P}\varphi$, the reason for this is that $\ulcorner \neg\varphi \urcorner$ under τ^{+} is not in $\mathscr{N}$ if $\mathrm{P}\varphi$, and, since the system is gapless, $\ulcorner \neg\varphi \urcorner$ must be put into $\overline{\mathscr{N}}$. So, there are two correspondences in a gapless system: 1. $\mathrm{O}\varphi$ corresponds to $\ulcorner \varphi \urcorner \in \mathscr{N}$, and 2. $\mathrm{P}\varphi$ ($\mathrm{P}\neg\varphi$) corresponds to $\ulcorner \neg\varphi \urcorner \in \overline{\mathscr{N}}$ ($\ulcorner \varphi \urcorner \in \overline{\mathscr{N}}$).

[5]If the norm-giver proclaims $\mathrm{P}\varphi$, then her/his intending and not proclaiming $\mathrm{O}\varphi$ would violate Gricean maxim of quantity, while the norm-recipient would be justified by the logic of cooperative communication to conclude that $\neg\mathrm{O}\varphi$ and therefore $\mathrm{P}\neg\varphi$.

3.1. Two types of inconsistency

"Relational (or external) inconsistency" occurs if $\mathscr{N} \cap \overline{\mathscr{N}} \neq \emptyset$. An example with $P\varphi \wedge O\neg\varphi$ is given in Figure 1. "Inner inconsistency" occurs if $\{\psi, \neg\psi\} \subseteq \overline{\mathscr{N}}$ for some ψ. An example with $O\psi \wedge O\neg\psi$ is given in Figure 1.

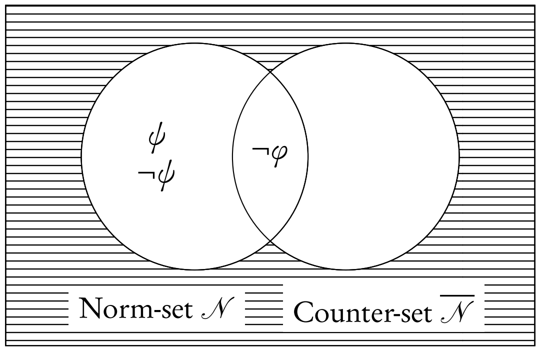

Figure 1: An example depicting a gapless but "doubly inconsistent" normative system. It is externally inconsistent since the intersection is non-empty, and internally inconsistent since the norm-set contains a pair of contradictory sentences.

Let us assume that norm-system $\langle \mathscr{N}, \overline{\mathscr{N}} \rangle$ actually has "relational perfections" that it ought to have:

- that it is gapless, $\mathscr{N} \cup \overline{\mathscr{N}} = \mathscr{L}$,
- that it is externally consistent, $\mathscr{N} \cap \overline{\mathscr{N}} = \emptyset$.

Figure 2 depicts a norm-system having the two relational perfections.

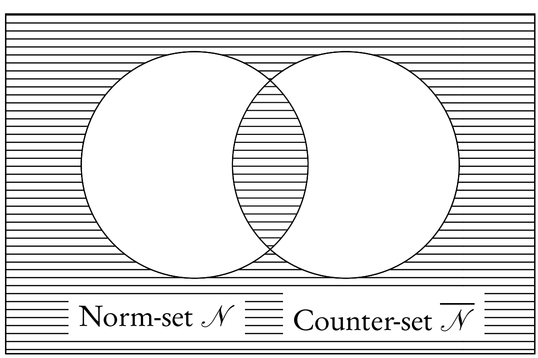

Figure 2: An example depicting a gapless and externally consistent system.

If the two relational perfections are present, translation function τ^{-} will give descriptions of properties of the counter-set that correspond to deontic postulates.

POSTULATE OF STANDARD DEONTIC LOGIC	NORM-SET PROPERTY	COUNTER-SET PROPERTY
(D) $\mathrm{O}\varphi \rightarrow \mathrm{P}\varphi$	consistency	completeness
(2) $(\mathrm{O}\varphi \wedge \mathrm{O}\psi) \rightarrow \mathrm{O}(\varphi \wedge \psi)$	closure under conjunction	having at least one conjunct for each conjunction contained
(K) axiom together with necessitation rule	deductive closure	"closure under implicant" (if φ entails ψ and $\psi \in \overline{\mathscr{N}}$, then $\varphi \in \overline{\mathscr{N}}$)

Table 1: The correspondence of set properties in a gapless and externally consistent norm-system.

Definition 3.1. Function $\tau^{-} : \mathscr{L}_{\mathrm{O}} \mapsto \mathscr{L}_{\mathscr{N}}$ translates formulas of the deontic language without iterated modalities to the language of the counter-set membership.

$$\tau^{-}(\varphi) = \varphi \qquad \text{if } \varphi \in \mathscr{L}_{pl}$$
$$\tau^{-}(\mathrm{O}\varphi) = \ulcorner\varphi\urcorner \notin \overline{\mathscr{N}}$$
$$\tau^{-}(\mathrm{P}\varphi) = \ulcorner\neg\varphi\urcorner \in \overline{\mathscr{N}}$$
$$\tau^{-}(\mathrm{P}\neg\varphi) = \ulcorner\varphi\urcorner \in \overline{\mathscr{N}}$$
$$\tau^{-}(\neg\varphi) = \neg\tau^{-}(\varphi)$$
$$\tau^{-}((\varphi \vee \psi)) = (\tau^{-}(\varphi) \vee \tau^{-}(\psi))$$

Consider some examples! (D) axiom, $\mathrm{O}\varphi \rightarrow \neg\mathrm{O}\neg\varphi$, characterizes *complete* counter-sets as shown by the translation: $\tau^{-1}(\mathrm{O}\varphi \rightarrow \neg\mathrm{O}\neg\varphi) = \ulcorner\varphi\urcorner \notin \overline{\mathscr{N}} \rightarrow \ulcorner\neg\varphi\urcorner \in \overline{\mathscr{N}}$. Adjunction axiom (2), $(\mathrm{O}\varphi \wedge \mathrm{O}\psi) \rightarrow \mathrm{O}(\varphi \wedge \psi)$, characterizes counter-sets having *at least one conjunct for any conjunction they contain* as shown by the translation (which makes use of the law of contraposition): $\tau^{-1}((\mathrm{O}\varphi \wedge \mathrm{O}\psi) \rightarrow \mathrm{O}(\varphi \wedge \psi)) = \ulcorner\varphi \wedge \psi\urcorner \in \overline{\mathscr{N}} \rightarrow (\ulcorner\varphi\urcorner \in \overline{\mathscr{N}} \vee \ulcorner\psi\urcorner \in \overline{\mathscr{N}})$. The counter-set property corresponding to (K) axiom is the property of being "downwardly" closed, i.e., closed under the affirmation of consequent, $\tau^{-}(\mathrm{O}(\varphi \rightarrow \psi) \rightarrow (\mathrm{O}\varphi \rightarrow \mathrm{O}\psi)) = \ulcorner\varphi \rightarrow \psi\urcorner \in \overline{\mathscr{N}} \rightarrow (\ulcorner\psi\urcorner \in \overline{\mathscr{N}} \rightarrow \ulcorner\varphi\urcorner \in \overline{\mathscr{N}})$.

The translations, summarized in Table 1, show the following fact: if the norm-set and the counter-set are identified with set of truth-sets of their respective elements, then the resulting pair of structures will consist of a filter, $\{[\![\varphi]\!] \mid \varphi \in \mathscr{N}\}$, and a weak ideal, $\{[\![\varphi]\!] \mid \varphi \in \overline{\mathscr{N}}\}$.

The social roles variedly relate to descriptive and normative properties of the norm-set. An actor's relation to a normative property can be conceptualized in terms of second-order norms. For example, the fact that consistency is a normative property (perfection property) for the norm-giver generates the second-order norm for the norm-giving activity: the norm-giver ought to build a consistent norm-set. Taking into account the two-sets model introduced here and the correspondence between perfection properties of the two sets, this line of thought should be extended in order to cover the permission-norm giving activity. So, the corresponding second-order norm with respect to the counter-set would be: the norm-giver ought to build a complete counter-set. But unlike consistency this property cannot be achieved since it would involve an infinite sequence of communicative-acts on the side of the permission-norm giver. So, is the completeness of the counter-set a normative property for the norm-giver or not? There are two ways for the norm-giver to achieve the completeness of the counter set: either by proclaiming that everything not forbidden is permitted or, vice versa, that everything not permitted is forbidden. This proclamation, or the "sealing legal principle" (Mastop 2011), would represent a meta-norm giving activity. In the two-sets model the sealing of the system can be performed in two ways. For the purpose of permitting the non-forbidden the following condition does the job: if $\ulcorner \varphi \urcorner \notin Cn(\mathscr{N})$, then $\ulcorner \varphi \urcorner \in \overline{\mathscr{N}}$. In the direction of forbidding the non-permitted the under-determinacy reigns and the resulting systems can be radically different.

4. Social pragmatics of inconsistency

The thesis of this paper is that second-order norms come in two types. The already discussed first type concerns the relation of the norm-giver and the norm-recipient towards the perfection properties of a norm-system in norm-giving and in normative reasoning. The second type is corrective and covers relations towards an imperfect norm-system. If the norm-giver has created an inconsistent normative system, then there is a "corrective obligation" for her/him: the norm-giver ought to restore consistency by the revision of its content (by derogations, modifications, ...). On the other hand, the norm-recipient ought to continue to reason on the basis of an inconsistent normative system, but it obviously cannot be done using classical logic where the principle *ex contradictione quodlibet* holds.[6] Therefore, in absence of other remedies, the second-order norm for the recipient's dealing with an inconsistent system requires the shift to an inconsistency-tolerant logic.[7] In other words, the

[6] In classical logic the presence of contradictory sentences in the premises destroys the proof by making every sentence provable, their presence in the theory destroys its descriptive power by making no interpretation possible, their presence in the discourse destroys communication by making it impossible to reach understanding. According to Tarski's (1956) theory of consequence relation ($Cn : \wp\mathscr{L} \mapsto \wp\mathscr{L}$) there must exist a sentence whose consequence is the whole of the language ($\mathscr{L}$). Tarski's Axiom 5 states this property as follows: *there is a sentence x such that* $Cn(\{x\}) = \mathscr{L}$.

[7] The requirement of changing logic seems to be a too radical solution, as has been noted by Jorge Rodriguez in the discussion at *A Workshop on Deontic Logic*, Milano. Instead of logic change he

norm-giver should revise the content and the norm-recipient should revise the logic of an inconsistent system, cf. Figure 3.

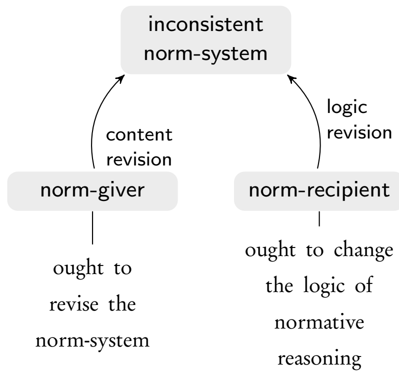

Figure 3: Corrective obligations with respect to an inconsitent normative-system.

The normative relation is different for each actor-role: the norm-giver ought to change the content of the normative system so to remove inconsistency, the norm-recipient ought to change the logic of in reasoning from the inconsistent normative system to its consequences. These second-order "dynamic" norms show the need of a notion of revision more general than that of AGM-revision (Alchourrón et al. 1985). The generalized notion of revision must be wide enough to encompass the case where revision includes no contraction but, instead, takes the form of the change of logic. Since a presence of contradiction in a set of sentences need not imply the absurdity of using this set in reasoning the two "imperfections" should be distinguished:

- the inconsistency, either internal $\{\psi, \neg\psi\} \subseteq \mathscr{N}$ or external $\mathscr{N} \cap \overline{\mathscr{N}} \neq \emptyset$, and
- the self-destruction ("anti-coherence") under logic L, where either $Cn(L, \mathscr{N}) = \mathscr{L}$ or $Cn(L, \mathscr{N} \cup \{\varphi\}) = \mathscr{L}$ for some φ of which $\mathrm{P}\varphi$ holds.[8]

4.1. Restoration and preservation conditions of logic revision

Let us make explicit, as has been done above, the logic defining particular consequence relation by introducing a place for it within the binary function $Cn(L, A)$.

has proposed a restricted type of normative reasoning in which the destructive effect is localized. Nevertheless, to change some of the "structural properties" of a consequence relation means to change the logic. It will be shown infra how the destructive effect of a contradiction can be localized while preserving consequence relations elsewhere within the deontic dialetheic logic.

[8]This means that $\neg\varphi \in \overline{\mathscr{N}}$ or, if $\varphi = \neg\psi$, then $\psi \in \overline{\mathscr{N}}$.

The notion of "anti-coherence" will be used here alongside the notion of consistency since some logics, such as dialetheic logic, are paraconsistent or "inconsistency tolerant" meaning that inconsistency of set does not entail its self-destruction or anti-coherence. In this case the revision cannot be understood in the way of AGM theory as consistency restoration. As previously stated this type of revision does not affect the content but only the form: it is the change of logic such that inconsistency will become divorced from anti-coherence. The revision without contraction can be understood as the change from the classical logic CL, where $\{\varphi, \neg\varphi\} \subseteq Cn(\mathsf{CL}, \mathscr{N})$ implies $Cn(\mathsf{CL}, \mathscr{N}) = \mathscr{L}$ to a non-classical logic NCL, where $Cn(\mathsf{NCL}, \mathscr{N}) \neq \mathscr{L}$. The consequence relation Cn in a logic L is a functional relation from a subset to a subset of the same language, $Cn : \wp\mathscr{L} \mapsto \wp\mathscr{L}$. The revision $\mathscr{N}[L/L^*]$ of a set $\mathscr{N}$ by changing logic from L to L^* is the deductive closure of the set under the logic $L^* : \mathscr{N}[L/L^*] = Cn(L^*, \mathscr{N})$.

The first condition that a logic change ought to satisfy is to restore a desirable logical property of the system being revised. In the first place, the change of logic ought to restore coherence of the set whose logic is being changed. If $\mathscr{N}$ is inconsistent and incoherent in classical logic, then $\mathscr{N}[\mathsf{CL}/\mathsf{NCL}]$ results in an inconsistent but coherent set, $\{\varphi, \neg\varphi\} \subseteq \mathscr{N}[\mathsf{CL}/\mathsf{NCL}]$ but $Cn(\mathsf{NCL}, \mathscr{N}) \neq \mathscr{L}$. Secondly, the change of logic ought to preserve desirable logical properties. Applying the preservation condition to the two-sets model of normative system, it is the perfection properties of the norm-set and the counter-set that ought to be saved. The two conditions of the logic revision, restoration condition and preservation condition, resemble the content contraction, but the difference lies in the fact that instead of consistency it is the coherence that is being restored, and, instead of maximal preservation of the content, it is the desirable logical properties that are being saved. So, the norm-recipient faced with an inconsistent normative system ought to adopt an inconsistency-tolerant logic under which the normative properties will be preserved, namely, closure under entailment and adjunction of the norm-set together with correlated properties of the counter-set (closure under implicants and closure under having at least one conjunct for each conjunction). Is there such a logic?

5. Dealing with normative inconsistency

As stated above, it is not the norm-giver's duty to change the logic, this is the norm-recipient's duty. The norm-recipient's duty remains the duty to treat the norm-system as a "logical object" (with the norm set closed under entailment and adjunction, and the counter-set having corresponding properties). But can this be done? Is there a logic not self-destructive in the case of inconsistency and still conservative with respect to "perfection properties" of the norm-set and counter-set? The answer is affirmative. Dialetheic deontic logic of Graham Priest (2006) both is inconsistency-tolerant and preserves properties of the norm-set and counter-set.

The notion of consistency of a normative-system is not the same notion as consistency of a theory. For example, the difference between external and internal consistency has no counterpart in the domain of descriptive theories. Theory as a set of sentences is semantically consistent if there is an interpretation that makes true every sentence in the set. This notion corresponds to internal consistency, i.e., consistency of the norm-set but, as will be shown below, the normative consistency is much more demanding than this. Let us further refine the set-theoretical model and treat $\mathscr{N}(w) \subseteq \mathscr{L}$ as a function that delivers obligations for situation w, and similarly for $\overline{\mathscr{N}}(w) \subseteq \mathscr{L}$. In this model the conditional obligation 'if φ, then $\mathrm{O}\psi$' is interpreted as 'if φ is the case in situation w, then $\ulcorner\psi\urcorner \in \mathscr{N}(w)$'. Since contraposition holds the conditional '$\mathrm{O}\psi$ only if φ' can be read as 'if $\neg\varphi$, then $\neg\mathrm{O}\psi$', and then, using the way of thinking connected with translation function τ^{-}, the following translation can be obtained: 'if $\neg\varphi$, then $\psi \in \overline{\mathscr{N}}$'. This approach reveals the highly demanding character of normative consistency: a normative system is strongly consistent if it provides a legal way out of any possible situation. The definition of strong consistency in the two-sets model $\langle \mathscr{N}, \overline{\mathscr{N}} \rangle$ follows.

Definition 5.1. Normative-functions system $\langle \mathscr{N}, \overline{\mathscr{N}} \rangle$ is strongly consistent iff for all w both $\mathscr{N}(w)$ is consistent and $\mathscr{N}(w) \cap \overline{\mathscr{N}}(w) = \emptyset$ holds.

Example 5.1. Suppose that the following holds: (i) if φ_1 is the case in a situation w, then $\ulcorner\psi\urcorner \in \mathscr{N}(w)$, and (ii) if φ_2 is the case in a situation w, then $\ulcorner\neg\psi\urcorner \in \mathscr{N}(w)$. Is it permitted for the norm-recipient to conclude that $\mathscr{N}$ has no normative force if the situation where $\varphi_1 \wedge \varphi_2$ holds is possible? Nevertheless, the norm-set function $\mathscr{N}$ might be inconsistent with respect to $\varphi_1 \wedge \varphi_2$-type of situations and consistent with respect to all other types of situations.

What is rational for a norm-recipient to infer regrading a normative-function system that is not strongly consistent? How far-reaching is the destructive power of contradiction: if the norm-set function $\mathscr{N}$ delivers an inconsistent set in w and a consistent set in any other case, does "anti-coherence" under classical logic at w, $\mathscr{N}(w) = \mathscr{L}$, give reason to abandon $\mathscr{N}$ at v where set $\mathscr{N}(v)$ is coherent? The same questions arise regarding the external consistency: does the fact of external inconsistency in some situation w (i.e., $\mathscr{N}(w) \cap \overline{\mathscr{N}}(w) \neq \emptyset$) give the reason to abandon the whole normative-functions system?

It is not only irrational to throw away a system because of its inconsistency in some possible situation, which as a matter of fact might never occur. It is also beyond the power of the norm-recipient to do so (unless the same person plays the role of the norm-giver and the norm-recipient). Rather, from the perspective of the norm-recipient the normative inconsistency arising in some situation shows that the normative system has entered its unstable phase, as depicted in Figure 4, and it is solely in these situations that the corrective obligation of changing the logic of normative reasoning applies to the norm-recipient. So, the norm-recipient must apply a special deontic logic that can function in an unstable phase. Is there a logic not self-destructive in the case of inconsistency and still conservative with respect to "perfection properties" of the norm-set and counter-set? The answer is affirmative. Dialetheic deontic logic of Graham Priest (2006) is both inconsistency-tolerant and preserves perfection properties of the norm-set and counter-set and therefore can function as the logic of normative instability.

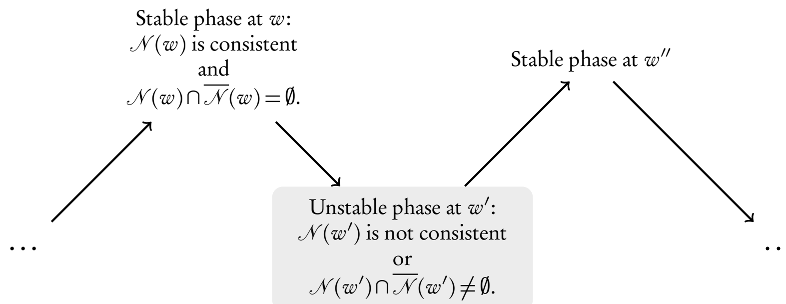

Figure 4: An exemplar sequence of phases of a normative system.

5.1. Dialetheic deontic logic

In this subsection a short presentation and a slight modification of the dialetheic deontic logic will be given and it will be shown how aptly it fits to the purpose of providing an inconsistency tolerant yet perfection properties preserving logic. The subsection ends with a short appendix addressing the question of expressive incompleteness of dialetheic deontic logic.

Priest (2006) presents the system of dialetheic deontic logic, a logic with three values. A contradiction within a system formulated in the language of dialetheic deontic logic does not cause its destruction. From the point of view of deontic pragmatics this is a desirable property since the norm-recipient as such does not have the power to revise the content of a norm-set. In an unstable phase the norm-recipient does not cease to be subordinated to the norm set. The second order obligation to reason on the basis of the normative system still holds and the only way to find a legal way out is to change its logic and not its content.

Dialetheic logic provides a natural extension of the semantic definitions of classical logic so that the expressions "is true" and "is false" are replaced by "has truth" and "has falsity". A sentence can have truth or falsity in itself, or both. Slightly departing

$\neg$	
{t}	{f}
{t,f}	{t,f}
{f}	{t}

$\wedge$	{t}	{t,f}	{f}
{t}	{t}	{t,f}	{f}
{t,f}	{t,f}	{t,f}	{f}
{f}	{f}	{f}	{f}

$\vee$	{t}	{t,f}	{f}
{t}	{t}	{t}	{t}
{t,f}	{t}	{t,f}	{t,f}
{f}	{t}	{t,f}	{f}

Table 2: The semantic definitions for the truth-functional (extensional) connectives in dialetheic logic.

from the notation of (Priest 2006) the three values of dialetheic logic will be denoted here by {t}, {t,f}, {f}.[9]

Priest's dialetheic semantics for truth-functional connectives (cf. Table 2) is easy to remember since its definitions for $\neg$, $\wedge$ and $\vee$ can be obtained from the classical definitions by replacing "α is true (false)" by "α has truth (falsity)", i.e., by replacing the values in the valuation v as follows: $v(\alpha) = \mathrm{t}$ and $v(\alpha) = \mathrm{f}$ by $\mathrm{t} \in v(\alpha)$ and $\mathrm{f} \in v(\alpha)$. One can also think of dialetheic valuations as a special case of "classical valuations" since negation inverts the order, conjunction picks the lowest value, and disjunction picks the greatest value of the two with respect to the given order:

$$\{\mathrm{f}\} < \{\mathrm{t,f}\} < \{\mathrm{t}\} \tag{order}$$

$$v(\neg\varphi) = \begin{cases} \{\mathrm{t,f}\} - v(\varphi) \text{ if } v(\varphi) \in \{\{\mathrm{t}\},\{\mathrm{f}\}\}, \\ \{\mathrm{t,f}\} \text{ otherwise.} \end{cases} \tag{$\neg$}$$

$$v(\varphi \wedge \psi) = min(v(\varphi), v(\psi)) \tag{$\wedge$}$$

$$v(\varphi \vee \psi) = max(v(\varphi), v(\psi)) \tag{$\vee$}$$

A contradiction ($\alpha \wedge \neg\alpha$) can be *both* true *and* false if and only if α is *both* true *and* false and $\neg\alpha$ is *both* true *and* false, cf. Table 3. The alleged slogan of dialethism states that "there are true contradictions", but this does not mean that there can be a contradiction that "bears no falsity on its face".

α	$\wedge$	$\neg\alpha$
{t}	{f}	{f}
{t,f}	{t,f}	{t,f}
{f}	{f}	{t}

Table 3: Possible values of a contradiction.

The dialetheic entailment $\alpha \rightarrow_d \beta$ with the respect to the real world is defined in (Priest 2006, 190) as it being the case that it is necessary (relative to the real world) that the truth is inherited from left to right and falsity form right to left.

[9] In order to preserve closeness to the natural language reading of the formulas the original expressions {1}, {1,0}, {0} of Priest have been replaced by {t}, {t,f}, {f} with the intended reading 'has truth only', 'has both truth and falsity', 'has falsity only'.

> …a necessary condition for entailment is truth preservation from antecedent to consequent. Truth preservation may not, on its own, be sufficient, however. For not only do we use the fact that something is entailed by true sentences to prove it, but we use the fact that something entails false ones to refute it. Thus, we require an entailment to preserve falsity from consequent to antecedent too. Of course, classically, truth preservation forwards and falsity preservation backwards go together. (Priest 2006, 84)

Priest extends the semantics to capture the meaning of the entailment operator $\rightarrow_d$. The initial language $\mathscr{L}_{\text{tf}}$ has the syntax $\alpha ::= p \mid \neg\alpha \mid (\alpha_1 \wedge \alpha_2)$, where $p \in \text{At}$ and At is the set of atomic sentences. The language $\mathscr{L}_{\text{tf}\rightarrow}$ has the syntax $\varphi ::= \alpha \mid (\varphi_1 \rightarrow_d \varphi_2)$, where $\alpha \in \mathscr{L}_{\text{tf}}$. An interpretation M is the tuple $\langle W, G, R, v\rangle$ where $W \neq \emptyset$, $G \in W$ with the intended meaning of G being the real world, $R \subseteq W \times W$, $\{G\} \times W \subseteq R$, v is a valuation function from $W \times \text{At}$ into $\{\{\text{t}\}, \{\text{t},\text{f}\}, \{\text{f}\}\}$. The shorthand notation for $v(w,\alpha)$ is $v_w(\alpha)$. The semantic definitions for truth-functional connectives are the same as in Table 2. The definition for the entailment connective is an intensional one:

- $\text{t} \in v_w(\alpha \rightarrow_d \beta)$ iff for all w' such that Rww':
  - if $\text{t} \in v_{w'}(\alpha)$, then $\text{t} \in v_{w'}(\beta)$, and
  - if $\text{f} \in v_{w'}(\beta)$, then $\text{f} \in v_{w'}(\alpha)$,
- $\text{f} \in v_w(\alpha \rightarrow_d \beta)$ iff for some w' such that Rww', $\text{t} \in v_{w'}(\alpha)$ and $\text{f} \in v_{w'}(\beta)$.[10]

(Priest 2006, 85) defines the semantic consequence relation as follows: "$\Sigma \models \alpha$ iff for all interpretations, M, it is true of the evaluation, v, that if $1 \in v_G(\beta)$ for all $\beta \in \Sigma$ then $1 \in v_G(\alpha)$". Modus ponens $\{\alpha, \alpha \rightarrow_d \beta\} \models \beta$ is not valid and since "modus ponens is a sine qua non of any implication connective" (Priest 2006, 86) Priest adds the condition that any world is accessible from G ("omniscience of G"), i.e. $\{G\} \times W \subseteq R$, which implies RGG and that is sufficient for validity of modus ponens.

Priest's semantic definition of the 'entailment conditional' ultimately reduces to left-to-right truth inheritance and right-to-left falsity inheritance in all valuations. The reason for this is that in examining the existence of consequence every model must be taken into account and, so, every valuation point must take the role of G

[10] In (Priest 2006, 85) the definition is given differently: instead of wRw' there appears $w'Rw$, which implies that the valuation v_w depends on valuations at worlds w' *from which* w is accessible, not those which are accessible from w. This non-standard definition seems to be a typographical error which reappears elsewhere but is not consistent with non-formal readings given in the text.

in some model; since every valuation point is accessible from G, the valuation of a conditional depends on all valuations. Therefore, for any valuation points w and w' it holds that if $w = G$ in M and $w' = G$ in M', then $v_w(\alpha \rightarrow_d \beta) = v_{w'}(\alpha \rightarrow_d \beta)$. The relational semantics of $\rightarrow_d$ with valuation function having two places, the one for a world and the other for a sentence, can be reduced to the test of validity of left-to-right truth inheritance and right-to-left falsity inheritance within the space of valuation semantics, where valuation is a one-place function. So, for the purpose of this text an alternative Definition 5.2 is proposed below.

Definition 5.2 (Alternative definition of $\rightarrow_d$)**.** The entailment conditional $\alpha \rightarrow_d \beta$ has truth in a valuation v, $\mathrm{t} \in v(\alpha \rightarrow_d \beta)$, iff for any valuation v' it holds that: (i) if $\mathrm{t} \in v'(\alpha)$, then $\mathrm{t} \in v'(\beta)$, and (ii) if $\mathrm{f} \in v'(\beta)$, then $\mathrm{f} \in v'(\alpha)$. The entailment conditional $\alpha \rightarrow_d \beta$ has falsity in a valuation v, $\mathrm{f} \in v(\alpha \rightarrow_d \beta)$, iff for some valuation v' it holds that: $\mathrm{t} \in v'(\alpha)$ and $\mathrm{f} \in v'(\beta)$.

With the basic notions at hand we can move to deontic dialetheic sentential logic. Priest gives the informal reading for $\mathrm{O}\alpha$ as follows: "…if the world were such that all extant obligations were duly fulfilled, then it would be the case that α" (Priest 2006, 189). The language domain in Priest (2006) is the whole propositional language, with truth-functional connectives and the intensional entailment operator $\rightarrow_d$, and, so, it includes, besides norm-contents in the proper sense, also the truths of logic. Since the content of proper norms is some doable state of affairs, the language under consideration can be reduced to its truth-functional part $\mathscr{L}_{\mathrm{tf}\rightarrow}$ and thus the following non-formal interpretation can be proposed. The extension $\omega^+(w) \subseteq \mathscr{L}$ covers the range of *obligatory* and *neccessary* sentences, sentences about that which is either obligatory or inevitable in the situation w. The anti-extension $\omega^-(w) \subseteq \mathscr{L}$ covers the range of *forbidden* and *optional* sentences, sentences about that which is either non-permitted or that which is permitted and negation of which is also permitted in the situation w. Priest takes that at any w the extension and anti-extension are exhaustive $\omega^+(w) \cup \omega^-(w) = \mathscr{L}_{\mathrm{tf}\rightarrow}$, but not necessarily exclusive.

Definition 5.3 (The syntax of the language $\mathscr{L}_{\mathrm{dd}}$ of dialetheic deontic logic.)**.** Let φ be a sentence of $\mathscr{L}_{\mathrm{tf}}$.

$$\alpha ::= \varphi \mid \mathrm{O}\varphi \mid \neg\alpha \mid (\alpha_1 \wedge \alpha_2). \quad (1)$$

$\mathrm{F}\varphi$ is an abbreviation for $\mathrm{O}\neg\varphi$. $\mathrm{P}\varphi$ is an abbreviation for $\neg\mathrm{O}\neg\varphi$.

The semantics of deontic dialetheic logic interprets the deontic operator O at a world w as the pair of extension $\omega^+(w)$ and anti-extension $\omega^-(w)$.

Definition 5.4. An extension $\omega^+(w) \subseteq \mathscr{L}_{\mathrm{tf}\rightarrow}$ at w is a set of sentences such that

- ω^+ is closed under dialetheic entailment: if α entails β and $\alpha \in \omega^+(w)$, then $\beta \in \omega^+(w)$,
- ω^+ is closed under adjunction: if $\alpha \in \omega^+(w)$ and $\beta \in \omega^+(w)$, then $(\alpha \wedge \beta) \in \omega^+(w)$.

Definition 5.5. An anti-extension $\omega^-(w) \subseteq \mathscr{L}_{\mathrm{tf}\rightarrow}$ at w is a set of sentences such that

- $\omega^-(w)$ is reversely closed under dialetheic entailment (closed under "affirmation of consequent"): if α entails β and $\beta \in \omega^-(w)$, then $\alpha \in \omega^-(w)$,
- $\omega^-(w)$ contains at least one conjunct for any conjunction it has: if $\alpha \wedge \beta \in \omega^-(w)$, then $\alpha \in \omega^-(w)$ or $\beta \in \omega^-(w)$.

Condition 1. The extension and anti-extension are exhaustive at any w: $\omega^+(w) \cup \omega^-(w) = \mathscr{L}_{\mathrm{tf}\rightarrow}$

Definition 5.6. Valuation $v_w(\mathrm{O}\varphi)$ of a sentence $\mathrm{O}\varphi$ in a situation w:

- $\mathrm{t} \in v_w(\mathrm{O}\varphi)$ iff $\varphi \in \omega^+(w)$,
- $\mathrm{f} \in v_w(\mathrm{O}\varphi)$ iff $\varphi \in \omega^-(w)$.

Example 5.2. The valuations for $\mathrm{F}\varphi$ and $\mathrm{P}\varphi$ follow from Definition 5.6 and matrices from Table 2. E.g., $\mathrm{t} \in v_w(\mathrm{P}\varphi)$ (iff $\mathrm{t} \in v_w(\neg\mathrm{O}\neg\varphi)$ iff $\mathrm{f} \in v_w(\mathrm{O}\neg\varphi)$) iff $\neg\varphi \in \omega^-(w)$; $\mathrm{f} \in v_w(\mathrm{P}\varphi)$ (iff $\mathrm{f} \in v_w(\neg\mathrm{O}\neg\varphi)$ iff $\mathrm{t} \in v_w(\mathrm{O}\neg\varphi)$) iff $\neg\varphi \in \omega^+(w)$. The comparison of the dialetheic definition with the set-theoretic translations for $\mathrm{P}\varphi$, $\ulcorner\neg\varphi\urcorner \notin \mathscr{N}$ and $\ulcorner\neg\varphi\urcorner \in \overline{\mathscr{N}}$, shows a structural similarity. If $\neg\varphi \notin \omega^+(w)$, then, thanks to exhaustivity of $\omega^+(w) \cup \omega^-(w)$, it must be the case that $\neg\varphi \in \omega^-(w)$ and that is the condition for $v_w(\mathrm{P}\varphi)$ to "contain the truth". On the other hand, if $\neg\varphi \in \omega^+(w)$, then this is the condition for $v_w(\mathrm{P}\varphi)$ to "contain the falsity".

It is obvious that Priest's "extension" and "anti-extension" not only correspond to the norm-set and counter-set, respectively, but also share their interconnected perfection properties with respect to closure under entailment and adjunction, cf. Table 1 supra. It is also clear that if each sentence is replaced with its truth-set, see Definition 5.8 and Proposition 5.5 below, then the resulting set of truth-sets for sentences from the extension will be a filter, while the resulting set of truth-sets for sentences from the anti-extension will be a weak ideal. In addition to this the relation between the two sets also has a perfection property, namley "gaplessness". It is only the perfection property of consistency that is lacking, or rather "consistencies" since there are two of them, the internal and external.

$w ::=$	SUBSETS OF $\omega^+(w) \cup \omega^-(w)$			VALUATION v_w			
	$\omega^+(w) - \omega^-(w)$	$\omega^+(w) \cap \omega^-(w)$	$\omega^-(w) - \omega^+(w)$	$O\varphi$	$F\varphi$	$P\varphi$	$P\neg\varphi$
w_1	$\varphi, \neg\varphi$			{t}	{t}	{f}	{f}
w_2	φ	$\neg\varphi$		{t}	{t,f}	{t,f}	{f}
w_3		$\varphi, \neg\varphi$		{t,f}	{t,f}	{t,f}	{t,f}
w_4		φ	$\neg\varphi$	{t,f}	{f}	{t}	{t,f}

Table 4: Varieties of inconsistent normative systems. Firstly, note that the presence of contradictory sentences in $\omega^-(w)$ is not a sign of inconsistency, but the indication of the fact that φ is optional. The sufficient conditions for inconsistency are the presence of contradictory sentences in $\omega^+(w)$ and $\omega^+(w) \cap \omega^-(w) \neq \emptyset$. Secondly, note the absence of "deontic explosion". In each of the inconsistency types it is not the case that "anything goes", not everything is permitted. In w_1 nothing is permitted with respect to φ. In w_2 it is only "partially permitted" that φ. In w_3 the inconsistency reaches its highest point and both φ and $\neg\varphi$ are simultaniously permitted and not permitted. In w_4 it is permitted that φ and permitted and not permitted that $\neg\varphi$, and, so, φ is both optional and not optional.

Example 5.3. The characteristic axiom schema of deontic logic is (D) $O\varphi \rightarrow P\varphi$. (D) axiom fails if the extension $\omega^+(w)$ is inconsistent. Consider for example interpretations depicted in Table 4! In w_1 it is obligatory but not permitted that φ since $v_{w_1}(O\varphi \wedge P\varphi) = \{f\}$. In w_2 the situation is similar but in a "weaker" sense, $v_{w_2}(O\varphi \wedge P\varphi) = min(\{t\}, \{t,f\}) = \{t,f\}$; if axiom (D) is read in the sense of dialetheic entailment, then it is invalidated in w_2 since there is no "preservation of falsity from consequent to antecedent". In w_3 the amount of "true contradictions" is the highest and φ is there both obligatory and not, both forbidden and not, both permitted and not. In w_4 φ is both non-optional and optional.

There are two types of inconsistency: (i) inconsistent $\omega^*(w)$ or, in the terminology of this article, internal inconsistency, e.g., in w_1, w_2, w_3 in Table 4, and (ii) non-empty intersection of $\omega^+(w)$ and $\omega^-(w)$ or external inconsistency, e.g. w_2, w_3, w_4 in Table 4. The linguistic construction of $\omega^+(w)$ is straightforward: if the norm-giver proclaims that $O\varphi$ for situation w, then $\varphi \in \omega^+(w)$. The construction of $\omega^-(w)$ is more complicated. A permission is read out of the counter-set, if $\neg\varphi \in \omega^-(w)$, then φ is permitted in w. If in addition to this it also holds $\varphi \in \omega^+(w)$, then φ is also not permitted in w, creating thus a deontic contradiction. The norm-giver's proclamation of a permission-norm $P\varphi$ ($P\neg\varphi$) for a situation w is equated with the addition of $\neg\varphi$ (φ) to the anti-extension.

Concluding remarks on deontic dialetheic logic It is stunning how accurately deontic dialetheic logic fits the need of logic revision on the side of the norm-recipient. The shift to deontic dialetheic logic in an unstable phase of the normative system fully satisfies restoration and preservation conditions of logic revision, cf. subsection 4.1. Nevertheless, there is a price for this and it takes the form of the increase of uncertainty. Proposition 5.4 below shows that the language of deontic dialetheic logic is not expressively complete (in the sense of Definition 5.10). The consequence of this fact is an unavoidable measure of uncertainty in normative reasoning. If the norm-recipient arrives at the conclusion φ it does not mean that any state $v \in [\![\varphi]\!]$ is acceptable since some of these might be excluded by other norms. The only way to solve the problem of safety in normative reasoning is to arrive at some sentence ψ which leaves no space for a possible defeat (with respect to a finite number of atomic descriptions). The truth-set of such a sentence is a singleton, $|[\![\varphi]\!]| = 1$, but this would be possible only in the case of the fully inconsistent state represented by the valuation $v^{\{t,f\}}$ which assigns both truth and falsity to any sentential letter, cf. Proposition 5.3. But this is exactly what a normative system does in its unstable phase of inconsistency: it forces the norm-recipient into a quandary.

5.2. Appendix

Concerning the language $\mathscr{L}_{tf}$

Definition 5.7. A valuation set W for a set of sentential letters At is the set of all functions from At to the set of semantic values $\{\{t\},\{t,f\},\{f\}\}$.

Definition 5.8. The intension $[\![\varphi]\!]^W$ of a sentence φ within a valuation set W is the set of valuations where $v(\varphi) = \{t\}$ or $v(\varphi) = \{t,f\}$: $[\![\varphi]\!]^W = \{v \in W \mid t \in v(\varphi)\}$.

Proposition 5.1. $[\![\varphi \wedge \psi]\!]^W = [\![\varphi]\!]^W \cap [\![\psi]\!]^W$; $[\![\varphi \vee \psi]\!]^W = [\![\varphi]\!]^W \cup [\![\psi]\!]^W$

Definition 5.9. Valuation $v_W^{\{t,f\}}$ is the valuation that assigns $\{t,f\}$ to all sentential letters.

Proposition 5.2. *For any sentence* φ, $v_W^{\{t,f\}}(\varphi) = \{t,f\}$.

Proof. By induction. In the basic case it holds for any sentential letter p that $v_W^{\{t,f\}}(p) = \{t,f\}$. In inductive step assume that less complex formulas φ and ψ satisfy the condition $v_W^{\{t,f\}}(\varphi) = \{t,f\}$ and $v_W^{\{t,f\}}(\psi) = \{t,f\}$. Then by the definitions of truth-functional connectives, $v_W^{\{t,f\}}(\neg\varphi) = \{t,f\}$, $v_W^{\{t,f\}}(\varphi \wedge \psi) = \{t,f\}$, $v_W^{\{t,f\}}(\varphi \vee \psi) = \{t,f\}$. □

Proposition 5.3. *Any sentence* φ *has* $v_W^{\{t,f\}}$ *in its intension* $[\![\varphi]\!]^W$.

Proof. Use Proposition 5.2 and Definition (5.8). □

Corollary 1. *For all* φ, *if* $\{v^{\{t,f\}}\} \neq [\![\varphi]\!]$, *then* $1 < |[\![\varphi]\!]|$.

Proof. Suppose that $\neg 1 < |[\![\varphi]\!]|$. Then either (i) $0 = |[\![\varphi]\!]|$ or (ii) $1 = |[\![\varphi]\!]|$. Both the first and the second are impossible since $v^{\{t,f\}}$ is any intension. □

Corollary 2. $[\![\varphi]\!] \cap [\![\neg\varphi]\!] \neq \emptyset$

Definition 5.10. The language $\mathscr{L}$ of a sentential logic is expressively complete with respect to the set W of valuations iff far any set $a \subseteq W$ of valuations there is a sentence $\varphi \in \mathscr{L}$ the intension of which is identical to the set $a = [\![\varphi]\!]$.

Proposition 5.4. *The language $\mathscr{L}_{\text{tf}}$ is not expressively complete with the respect to the set W of valuations of dialetheic semantics.*

Proof. Use Corollary 2. □

Concerning the language $\mathscr{L}_{\text{tf}\rightarrow}$

Lemma 5.1. *For any v and v', $v(\varphi \rightarrow \psi) = v'(\varphi \rightarrow \psi)$.*

Proof. It has to be proved that for any v and v', if $\text{t} \in v(\varphi \rightarrow \psi)$, then $\text{t} \in v'(\varphi \rightarrow \psi)$, and if $\text{f} \in v(\varphi \rightarrow \psi)$, then $\text{f} \in v'(\varphi \rightarrow \psi)$. This is an immediate consequence of Definition 5.2. □

Proposition 5.5. $[\![\varphi \rightarrow \psi]\!] = W$ *or* $[\![\varphi \rightarrow \psi]\!] = \emptyset$.

Proof. From Lemma 5.1 it follows that if there is a valuation v such that $v \in [\![\varphi \rightarrow \psi]\!]$, then for any valuation v it holds that $v \in [\![\varphi \rightarrow \psi]\!]$. So, if there is a v such that $v \in [\![\varphi \rightarrow \psi]\!]$, then $[\![\varphi \rightarrow \psi]\!] = W$. If for some v, $v(\varphi \rightarrow \psi) = \{\text{f}\}$, then, by Lemma 5.1, $v'(\varphi \rightarrow \psi) = \{\text{f}\}$ for any v'. Therefore, $[\![\varphi \rightarrow \psi]\!] = \emptyset$. □

Proposition 5.6. *If $v_w(p) = \{\text{t}, \text{f}\}$ for all propositional letters, then for any $\varphi \in \mathscr{L}_{\text{tf}\rightarrow}$, if $[\![\varphi]\!] \neq \emptyset$, then $v_w(\varphi) = \{\text{t}, \text{f}\}$.*

Proof. Use induction. The basic case $\varphi \in \mathscr{L}_{\text{tf}}$ holds by Proposition 5.2. Assume inductive hypothesis: $v_w(\varphi) = \{\text{t}, \text{f}\}$ and $v_w(\psi) = \{\text{t}, \text{f}\}$. Consider $\varphi \rightarrow_d \psi$. From Definition 5.2 and inductive hypothesis it follows that $\text{f} \in v_x$ for any $x \in W$, i.e., $\text{t} \in v_w(\varphi)$ and $\text{f} \in v_w(\psi)$. $[\![\varphi \rightarrow \psi]\!] = W$ from Proposition 5.5 and the main assumption. So, on one side, as established, $\text{f} \in \text{v}_x(\varphi \rightarrow \psi)$ for any $x \in W$, and, on the other side, $\text{t} \in v_x(\varphi \rightarrow \psi)$ for any $x \in W$ by the Definition 5.8. Then, by universal instantiation, $v_w(\varphi \rightarrow \psi) = \{\text{t}, \text{f}\}$ The cases for $\neg$, $\wedge$ and $\vee$ are similar as in the proof of Proposition 5.2.

□

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