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ON PROBABLE CONDITIONALS

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ABSTRACT

We compare supports $A \uparrow B$ (i.e. $pr(B|A) > pr(B)$) with conditionals $A \to B$ and prove that the basic properties of supports are exactly the opposite to those of conditionals.

Keywords: probable conditionals

We are interested in probable conditionals which could be $pr(A \to B)$ "*probability of, B if A*" or $pr(B|A)$ "*probability of B, if A*".

Lewis famously proved that they are not the same; what he assumes about the conditionals is evident from his proof, (cf. Lewis pp. 297-230):

If $pr(A \to B)$ is the same as $pr(B|A)$ then

$$
\begin{aligned}
pr(B|A)& \\
&= pr(A \to B)\\
&= pr(A \to B|B)pr(B)\\
&\qquad + pr(A \to B|-B)pr(-B)\\
&= pr(B \to (A \to B))pr(B)\\
&\qquad + pr(-B \to (A \to B))pr(-B)\\
&= 1pr(B)\\
&\qquad + pr((-B\&A) \to B)pr(-B)\\
&= 1pr(B)\\
&\qquad + pr(B|-B\&A)pr(-B)\\
&= 1pr(B) + 0pr(-B)\\
&= pr(B)
\end{aligned}
$$

Hence, for every A and every B we have $pr(B|A) = pr(B)$ and this is a contradiction. The whole point is that $pr(B \to -B) = pr(-B)$ could be positive, whereas $pr(-B|B)$ is necessarily zero.

For classical conditional $\rightarrow$ we prove the same (even more elementary):

$$\begin{aligned} pr(A \rightarrow B) &= pr(-A \vee B) \\ &= pr(-A \vee AB) \\ &= pr(-A) + pr(AB) \\ &= pr(-A) + pr(A)pr(B|A) \\ &= 1 - x + xc, \end{aligned}$$

where $x = pr(A)$ and $c = pr(B|A)$. Hence $pr(A \rightarrow B) = pr(B|A)$, i.e. $1-x+xc = c$, only if $pr(A) = x = 1$ or $pr(B|A) = c = 1$, i.e. if $pr(A \rightarrow B) = pr(B|A) = 1$.

Yet more elementary than that, take $S = E \rightarrow T =$ If *"it is even on the dice"*, then *"it is two on the dice"*. Then $pr(-(E \rightarrow T)) = pr(\text{even \& not two}) = 1/3$, i.e. $pr(E \rightarrow T) = 2/3$, but $pr(T|E) = 1/3$.

As a matter of fact, it is easy to prove that (for classical conditional $\rightarrow$)
$pr(A \rightarrow B) \geq pr(B|A)$ for every A and B:

$$\begin{aligned} pr(A \rightarrow B) &= pr(-(A(-B))) \\ &= 1 - pr(A(-B)) \\ pr(B|A) &= pr(BA)/pr(A) \\ &= (pr(A) - pr(A(-B)))/pr(A) \\ &= 1 - pr(A(-B))/pr(A) \end{aligned}$$

But $pr(A(-B)) \leq pr(A(-B))/pr(A)$ (because $pr(A) \leq 1$) and then $1 - pr(A(-B)) \geq 1 - pr(A(-B))/pr(A)$ i.e. $pr(A \rightarrow B) \geq pr(B|A)$, which was to be proved.

It also fallows that $pr(A \rightarrow B) = pr(B|A)$ only if $pr(A) = 1$ or $pr(A(-B)) = 0$ i.e. $pr(-(A(-B))) = 1$ i.e. $pr(A \rightarrow B) = 1$ (in accordance with what we have already proved).

Another kind of probable conditional is $A \uparrow B$, which means *"A makes B more probable" ("A supports B")* and which is defined as $pr(B|A) > pr(B)$. We could also define $A \downarrow B$, which means *"A makes B less probable"* (*"A subverts B"*) as $pr(B|A) < pr(B)$. The independence relation $A \perp B$, is defined as $pr(B|A) = pr(B)$.

For lot of people it is tempting to transfer the properties of conditionals to properties of supports. This is quite a common error.

People think: "if A supports B and B supports C, then A supports C" (i.e. they think of "supports" as transitive). Confronted with a concrete counterexample: $A =$*"having white hair"*, $B =$*"being over 50"*, $C =$ *"being completely bald"*, they change their minds.

People also think: "If A supports C and B supports C, then their conjunction does so even more". Confronted with a concrete counterexample: A crime is committed by two men, one in a red jacket another in a black coat. $A =$ *"first witness recognized the suspect as the man in the red jacket"*, $B =$ *"second witness recognized the (same) suspect as the man in the black coat"*, $C =$ *"the suspect is guilty"*, they change their minds. There are other concrete counterexamples in ch. 6. of Carnap's Logical *Foundations of Probability.*

It seems that people do not err in concrete contexts and do err in abstract contexts. There is a lot of psyhological research on this topic. Perhaps the most famous is the *Wason selection task* (cf. Wason). People have a hard time solving it in abstract contexts but can usually solve it correctly in concrete contexts, in particular when the concrete context is policing a social rule (cf. Cosmides, Tooby). This is the reason that an abstract analysis of supports $\uparrow$ (compared to conditionals $\rightarrow$) could be of some interest.

The basic properties of conditionals are:

(1) $A \rightarrow B, B \rightarrow C \Rightarrow A \rightarrow C$ is valid (transitivity)

(2) $A \rightarrow B \Rightarrow B \rightarrow A$ is not valid (converse fallacy)

(3) $A \rightarrow B \Rightarrow -B \rightarrow -A$ is valid (contraposition)

(4) $A \rightarrow B \Rightarrow -A \rightarrow -B$ is not valid (inverse fallacy)

(5) $C \rightarrow A, C \rightarrow B \Rightarrow C \rightarrow A\&B$ is valid (conjunction introduction)

(6) $C \rightarrow A(C \rightarrow B) \Rightarrow C \rightarrow A \vee B$ is valid (disjunction introduction)

(7) $A \rightarrow C(B \rightarrow C) \Rightarrow A\&B \rightarrow C$ is valid (conjunction elimination)

(8) $A \rightarrow C, B \rightarrow C \Rightarrow A \vee B \rightarrow C$ is valid (disjunction elimination)

The corresponding properties of $A \uparrow B$, except (3), are exactly the opposite:

(i) Property (2) is valid for $\uparrow$ (the relation is symmetrical).

(ii) Property (4) is valid for $\uparrow$.

(iii) Property (1) is not valid for $\uparrow$ (the relation is not transitive).

(iv) Properties (5), (6), (7) and (8) are not valid for $\uparrow$.

It is easy to prove that:

(i) The symmetry of $\uparrow$ and $\downarrow$ follows from $pr(A|B)pr(B) = pr(B|A)pr(A)$.

Namely, if $B \uparrow A$ i.e. $pr(A|B) > pr(A)$ but it is not the case that $A \uparrow B$ i.e. $pr(B|A) \leq pr(B)$ then $pr(A|B)pr(B) > pr(B|A)pr(A)$ and we have a contradiction with $pr(A|B)pr(B) = pr(B|A)pr(A)$.

Similarly, if $B \downarrow A$ i.e. $pr(A|B) < pr(A)$ but it is not the case that $A \downarrow B$ i.e. $pr(B|A) \geq pr(B)$ then $pr(A|B)pr(B) < pr(B|A)pr(A)$ and we again have a contradiction with $pr(A|B)pr(B) = pr(B|A)pr(A)$.

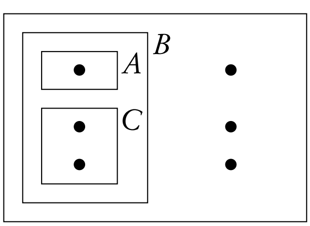
(ii) It is easy to prove that $A \uparrow B$ iff $A \downarrow -B$ and $A \downarrow B$ iff $A \uparrow -B$.

From $A \uparrow B$ i.e. $pr(B|A) > pr(B)$, it follows that $1 - pr(B|A) < 1 - pr(B)$ i.e. $pr(-B|A) < pr(-B)$ i.e. $A \downarrow -B$.

From $A \downarrow B$ i.e. $pr(B|A) < pr(B)$, it follows that $1 - pr(B|A) > 1 - pr(B)$ i.e. $pr(-B|A) > pr(-B)$ i. e. $A \uparrow -B$.

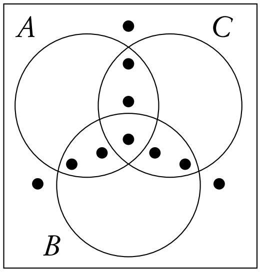
Now, using (i), it is easy to prove (4) for $\uparrow$ and $\downarrow$:
$A \uparrow B$ iff $A \downarrow -B$ iff $-B \downarrow A$ iff $-B \uparrow -A$ iff $-A \uparrow -B$
$A \downarrow B$ iff $A \uparrow -B$ iff $-B \uparrow A$ iff $-B \downarrow -A$ iff $-A \downarrow -B$

(iii) Nontransitivity of $\uparrow$ follows from the following example:



From the above figure it follows that $pr(B|A) = 1$ and $pr(B) = 1/2$ i.e. $A \uparrow B$. Similarly, $pr(C/B) = 2/3$ and $pr(C) = 1/3$ i.e. $B \uparrow C$. But $pr(C/A) = 0$ and $pr(C) = 1/3$ i.e. $A \downarrow C$. Hence, $\uparrow$ is not transitive. (It is easy to construct similar examples which prove the nontransitivity of $\downarrow$ and $\perp$.)

(iv) That (5) - (8) are not valid for $\uparrow$ follows from the following example:



From the above figure it follows that $pr(A|C) = 3/5$ and $pr(A) = 1/2$ i.e. $C \uparrow A$. Similarly, $pr(B|C) = 3/5$ and $pr(B) = 1/2$ i.e. $C \uparrow B$. But $pr(A\&B|C) = 1/5$ and $pr(A\&B) = 3/10$ i.e. $C \downarrow (A\&B)$. This proves that (5) is not valid for $\uparrow$.

It also follows that $pr(C|A) = 3/5$ and $pr(C) = 1/2$ i.e. $A \uparrow C$. (Similarly, $pr(C|B) = 3/5$ and $pr(C) = 1/2$ i.e. $B \uparrow C$.) But $pr(C|A\&B) = 1/3$ and $pr(A\&B) = 1/2$ i.e. $(A\&B) \downarrow C$. This proves that (7) is not valid for $\uparrow$.

From nonvalidity of (5) for $\uparrow$, it follows that

$$-C \uparrow -A, -C \uparrow -B \Rightarrow -C \uparrow (-A\& -B)$$

is not valid. Then it follows from (ii) that $C \uparrow A, C \uparrow B \Rightarrow C \uparrow -(-A\& -B)$ is not valid. Hence, $C \uparrow A, C \uparrow B \Rightarrow C \uparrow (A \vee B)$ is not valid. This proves that (6) is not valid for $\uparrow$.

From the above proof concerning (7) for $\uparrow$, it follows that

$$-C \uparrow -A, -C \uparrow -B \Rightarrow -C \uparrow (-A \vee -B)$$

is not valid. Then it follows from (ii) that $C \uparrow A$, $C \uparrow B \Rightarrow C \uparrow -(-A \vee -B)$ is not valid. Hence, $C \uparrow A$, $C \uparrow B \Rightarrow C \uparrow (A\&B)$ is not valid. This proves that (8) is not valid for $\uparrow$.

(It is easy to construct similar examples which prove that (5) - (8) are not valid for $\downarrow$ and $\perp$.)

REFERENCES

Carnap, R. 1950. *Logical Foundations of Probability*. University of Chicago Press.

Cosmides, L. and J. Tooby 1992. Cognitive Adaptions for Social Exchange. In *The Adapted Mind: Evolutionary Psychology and the Generation of Culture*, eds. J. Barkow, L. Cosmides and J. Tooby, pp. 163–228. Oxford University Press.

Lewis, D. 1976. Probabilities of conditionals and conditional probabilities. *The Philosophical Review* 85(3):297–315.

Wason, P. C. 1968. Reasoning about a rule. *Quarterly Journal of Experimental Psychology* 20(3): 273–281.

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