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BRIDGING TWO GULFS: HERMANN WEYL

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ABSTRACT

Professor Dummett challenged us to attempt to bridge two gulfs in communication, the first between contemporary philosophers and physicists, and the second between 'analytic' and 'continental' philosophy. Deeply sympathetic to the reasons motivating Dummett's challenge, I adduce the example of Hermann Weyl and his profound philosophical engagement with the mathematics and theoretical physics of his time. I also suggest that Weyl's epistemology and metaphysics of science stand in stark contrast to the default realism of much contemporary philosophy of physics, an attitude I regard as an obstacle to be overcome in furthering communication among philosophers and between philosophy and the wider intellectual culture.

Keywords: Weyl, general relativity, realism, Schlick, Reichenbach, Quine, Kuhn, gauge invariance, quantum mechanics, transcendental idealism, symmetry, constructive cognition

1. Introduction

I count myself as among those who remain personally indebted to Professor Dummett's pioneering efforts to illuminate the origins of the division between analytic and 'continental' (or, to use his more appropriate term 'synthetic') philosophy. In recent works (Ryckman, 2007; Friedman and Ryckman, 2010), I've endeavored to follow Dummett's lead in locating the pivotal moment in the differing philosophical legacies of Frege and Husserl, key figures that, as Dummett (1993, 26) observed, the typical German student of philosophy in, say, 1903, may well have regarded as remarkably similar in interests and outlook. Epitomizing the subsequent history in a memorable metaphor, Dummett noted that the respective influences of Frege, the "grandfather of analytic philosophy", and Husserl, a patriarch of "continental philosophy", run through 20th century philosophy like the Rhine and Danube, mighty rivers rising close together, briefly running parallel but then diverging to widely separate seas. With Dummett, I see the rift as both unnecessary and unfortunate, rooted not in intrinsic and irresolvable philosophical differences but in the contingent catastrophic political events of Europe in the first half of the

20th century. And I agree that the division continues to be maintained largely for sociological reasons, a matter deleterious to the health of contemporary philosophy. However, my disquiet with the analytic/continental divide is more narrowly targeted than Dummett's, since mine stems from ongoing reflection upon the trajectory given to 20th century philosophy of science by logical empiricism by the 1930s.

Here too occurred a pivotal episode, Einstein's theory of general relativity, then a corresponding radical divergence in ensuing philosophical tendencies. However, unlike the familiar division between analytic and continental wings where both tendencies survive yet fail to communicate, this development resulted – if one judges from the subsequent appearance of the discipline of philosophy of science – in near unconditional surrender of the terrain of philosophical discourse about science to logical empiricism, i.e., to analytic philosophy. Long after the demise of logical empiricism, the consequences of its triumph in the 1930s are still with us, contributing not merely to the breakdown of communication between philosophers and the educated lay public, but leading directly, as I see it, to an often scientistic and uncritically realist conception of contemporary science. One result is an impasse in contemporary philosophy of physics, where the default position is that of a practitioner of "naturalistic metaphysics", addressing "the foundational question par excellence", viz., "how could the world possibly be the way this theory says it is?"[1] According to this attitude, our best physical theories, rather like postcards from eternity, are to be considered "approximately true", and so tell us a highly reliable, if perplexing, story about the ontological furnishings of the world's deep structure. The vicissitudes and contingencies of the particular contexts in which such theories germinate, develop and are articulated, details situating theories as cultural products within a local scientific milieu, find no place in this conception of philosophy of physics' foundational quest. After all, the aim of metaphysics, however naturalized, remains the formation of true beliefs regarding the ultimate constituents of the world.

Let us recall that in the immediate post-WWI era, Einstein's theory, following the dramatic announcement in London of its empirical confirmation in November, 1919, quickly became an affair of enormous interest to the wider culture, comparable in impact only to the advent of Newton's theory of gravitation and the Darwin-Wallace theory of natural selection. Naturally, it became a principal focus of philosophical attention and inquiry. However, its abstract statement in the mathematics of the tensor calculus, not to mention the very fluidity of physical and mathematical meaning attending its fundamental principles, of equivalence, of general relativity, of general covariance, and, finally, of what Einstein (1918) termed "Mach's Principle", greatly complicated the task of coming to a synthetic understanding. Such initial ambiguities are not unusual in the history of physics. Many new theories bring unfamiliar mathematics, and physical theories, if sufficiently robust, are rarely, if ever, without

1 (van Fraassen, 1991, 4): "When we come to a specific theory, the question: how could the world possibly be the way this theory says it is? … is the foundational question par excellence, and it makes equal sense to both realist and empiricist alike." In fairness to van Fraassen's own antipathy to metaphysics, such beliefs need not be thought as extending beyond a overarching confidence in the theory's empirical adequacy, its ability to "save the phenomena" in its domain of application. But non-realism remains a minority viewpoint in contemporary philosophy of physics.

unproblematic aspects; indeed, they are often taken to say different things at different times. In this situation, it is understandable that there was considerable interpretive latitude for inherently antagonistic philosophical viewpoints, all seeking vindication, confirmation, or illumination from the revolutionary theory. Naturally, the very project of seeking to identify such a revolutionary transformation of thought with any one philosophical viewpoint is suspect from the outset, ignoring the fact that schools of philosophical interpretation in turn "evolve" to accommodate or domesticate the prestigious novel conceptions. This is precisely what happened in the case of general relativity.

Logical empiricism forthrightly admitted the influence of the theory of relativity in shaping the fundamental core of its outlook. Since the rise of logical empiricism in the late 1920s, from which stem, if only critically, the subsequent main trends in 20th century philosophy of science, it has been widely if not universally accepted that relativity theory had convincingly demonstrated the untenability of any species of Kantian-inspired philosophy of science.[2] If individual responsibility can be assigned for this assessment, it belongs to Moritz Schlick, to become the éminence grise of the Vienna Circle and logical empiricism generally, who, by 1921 had declared, publicly and prominently, that Einstein's gravitational theory decisively refuted all forms of transcendental idealism, or, as Schlick termed it, "the philosophy of the synthetic *a priori*".

Examination of the acuteness of Schlick's portrait of the rejected transcendental philosophy must be left aside here.[3] It may suffice to say that Schlick's alternative, a precursor of a contemporary form of scientific realism, rested on the conviction that "[t]hought never creates the relations of reality; it has no form which could imprint it, and reality allows no imprinting, for it is already formed". (Schlick 1918, 326-7; 309) Transcendental idealists, according to Schlick, had confused the "conceptual wrapping" of reality (in physical theory) with reality itself. Yet his realism about scientific theories had been worked out only schematically. In brief, Schlick assumed a purely formal axiomatic view of mathematical theories associated with the name of Hilbert, although this was but a misrepresentative slice of Hilbert's considerably more nuanced view of the nature of mathematics and its relation to physics. (Sieg 1999; Brading and Ryckman 2009) But as Schlick recognized, a formalist view of mathematical theories raises anew the problem of understanding the application of mathematics to empirical phenomena in mathematical natural science. Overlooking interesting details of Schlick's own philosophical evolution and his influence on his younger colleague Hans Reichenbach, Schlick concluded that the application of formal mathematical theory in physics is accounted for by "coordinations" or mappings, assigning mathematical structures to physical objects and concrete empirical phenomena. In this way, a mathematical theory in physics acquires empirical meaning. Through such coordinations, the formal structure applies literally to the world, the mathematical expressions are projected, as it were, onto the world itself either designating or defining the objects of physical theories.

2 At least in Anglo-American philosophy of science. Certainly post-WWII European traditions in philosophy of science have been far more catholic in sources and approach.

3 For a critical account, see (Ryckman 2005, chapter 3).

While Schlick's realism did not survive the anti-metaphysical purge of subsequent logical empiricism, all the pieces remained in place for what Schlick subsequently termed a "consistent empiricism", according to which the cognitive significance of statements, in particular of a scientific theory, lay either in definitional (and so analytic) relations among its logical and mathematical terms, or else in a posteriori empirical observations that confirm or disconfirm the formal expression. In particular, the cornerstone of logical empiricism, anchoring all the rest, declared that there are no synthetic *a priori* principles.[4] Any attribution of a non-analytic, non-empirical, source of mathematical and mathematical-physical cognition was deemed "meaningless" or, more derisively, "metaphysics".

In consequence, a principal result of the logical empiricist treatment of mathematical natural science was the complete obliteration of the transcendental problem of constitution of the object of knowledge. As we have seen, according to Schlick, the Kantian doctrine, of positing structures in experience not derived therefrom but regarded as the active contribution of the mind to experience, confused the conceptual wrapping of reality with reality itself. Although transcendental idealists maintained that the object-constituting role of mathematics in physical theory could neither be subsumed under, nor identified with, a coordination of mathematical conceptions to empirical phenomena, logical empiricist philosophy of science collapsed the former issue into the latter. The decisive step was taken in the "constitution theory" (Konstitutionstheorie) in Carnap (1928) where the new logic of Whitehead and Russell replaced Kant's transcendental logic, and all mention of "acts of constitution" is to be replaced by logical constructions from elementary experiences. In the Aufbau, "constitution of the object of knowledge" meant a chain of logical definitions within the type-theory of Principia Mathematica leading (in principle) from a single basic relation between elementary experiences to the reconstructed object in question.

Subsequently, the "scientific philosophy" recognized no constitutive function at all: Cognitively meaningful statements are either statements of logic and mathematics, which are analytic, or else synthetic a posteriori empirical statements, whose meaning resides in their translation into observational terms. As we know, Quine's attack on the logical empiricist analytic/synthetic distinction and Kuhn's critique of the methodology of rational reconstruction set the stage for much of philosophy of science in the second half of the 20th century. Yet neither Quine nor Kuhn appreciably widened the explanatory space for understanding how contemporary mathematical sciences of nature are at all possible; a persisting legacy of logical empiricism is that such questions belong to an earlier and mistaken conception of epistemology. Instead, philosophical inquiry posed another task, of explaining the success of many actual 20th century scientific theories in providing an increasingly unified, empirically accurate account of the phenomena of nature. By the end of the 1960s, as the unviability of logical empiricism's account of scientific theories became generally recognized, the dominant reaction to Kuhn's account of scientific revolutions was a return to scientific realism as

4 This is explicit in the "manifesto" of logical empiricism, Die Wissenschaftliche Weltanschauung (1929), written largely by Otto Neurath.

a bulwark against "relativism" and "irrationalist" accounts of scientific change. Hence, though contemporary scientific realism has faced significant criticism, it remains a default assumption in much of philosophy, science, and the wider intellectual culture.

However rhetorically useful, the claim that general relativity sounded the death knell of "the Kantian position" follows only if, as Schlick did, one ignored important post-Kantian developments of Kant's thought as well as many of the most significant developments in relativity theory in the period 1915-1925. Thus Schlick's judgment was narrowly based and by no means universally shared. To sample but one countering opinion, the Nobel prize winner, and fellow Planck student, Max von Laue stated, in the first actual textbook on general relativity, that Kantian epistemology was confirmed by the new theory, although "not every sentence of The Critique of Pure Reason" could be regarded as sacrosanct (von Laue 1921, 43). Yet as pious children of this world, to borrow an expression of Hermann Weyl's, we know that if an assertion is repeated sufficiently often, while remaining unchallenged in the forum of debate, it commonly enters into currency as accepted background knowledge. Certainly the claim is strewn throughout the literature on logical empiricism, percolating beyond to its prodigal progeny. Nor was it explicitly challenged in philosophical circles by anyone having the gravitas of authority possessed by Schlick, and then by Reichenbach who would take over the mantle of expertise on relativity theory within logical empiricism, as Schlick fell under the influence of Wittgenstein and turned away from philosophical investigations of physics. As a result, the allegation that general relativity falsified any manner of Kantian philosophy, as well as a myth that logical empiricism was completely d'accord with Einstein and with the leading theories of modern physics remained unimpeached amidst the triple assault that proved fatal to the rest of logical empiricism: Quine (and Tarski's) attack on the analytic-synthetic distinction, Hanson's and Toulmin's on the observational-theoretical distinction, and Kuhn's critique of logical empiricism's account of intertheory relations, and method of rational reconstruction.

Though hidden from view by the 1960s, alternatives to logical empiricism already existed, assuming several different but related forms, in the period between the world wars. Here I wish to concentrate on Hermann Weyl, articulating the most significant alternative, for three reasons relevant to the challenge presented by Dummett's essay. First, although trained as a mathematician, not a philosopher, Weyl's seminal contributions to 20th century theoretical physics as well as his expressly philosophical writings place him, perhaps uniquely, on center stage in any discussion of attempts to bridge the gulf in communication between contemporary physics and philosophy. Secondly, no other 20th century scientist of Weyl's stature (a miniscule set) invested anything like the effort and industry Weyl demonstrated in reaching out to the educated lay public in conveying his sense of the philosophical tendencies and directions of modern physics and mathematics. Finally, Weyl looms disproportionally large in bridging the second gulf Dummett identified, that between analytic and continental philosophy, simply by virtue of the fact that he wrote, as we shall see, more like a 'continental' than an 'analytic' philosopher about mathematics and science, matters almost invariably considered as within the exclusive purview of analytic philosophy. Admittedly a rara avis, Weyl nonetheless stands before us as an exemplar of philosophical engagement at the highest level with theoretical physics, mathematics, and natural science more generally. One would be hard pressed indeed to name another scientist, before or since, who has thought more deeply about the philosophical significance and implications of modern physics and mathematics or has been more concerned to transmit his thoughts to the wider culture of the educated non-scientific laity. Despite the remarkable scientific developments in the half century since Weyl's death in 1955, I believe that Weyl's example remains an instructive one to philosophers who, as I do, share Dummett's concern with the place of philosophy in European culture.

2. Why Weyl?

While perhaps only dimly if at all known to most philosophers, the name of Hermann Weyl needs no introduction to physicists or to historians and philosophers of physics cognizant of such concepts as Weyl spinor fields, the Weyl equation (for a massless Dirac field), the Weyl (integrated) form of the Heisenberg commutation relations, the Weyl tensor in Riemannian geometry and general relativity and the related Weyl curvature hypothesis (both baptized, I believe, by Roger Penrose). From John Wheeler and, much later, Norman Sieroka, we have learned that Weyl's speculations in the early 1920s on the topological origins of matter led to the concept of wormholes in space-time. Thanks to the efforts of Frank Yang, Lochlainn O'Raifeartaigh, Erhard Scholz, and Katherine Brading, many physicists and historians of physics are now aware that in 1918 Weyl, attempting to unify gravitation and electromagnetism, developed the notion of gauge invariance (originally as a local scale or conformal symmetry) and spelt out its relation to Emmy Noether's second theorem (pertaining to infinite parameter continuous groups) on invariants of Lagrangian systems. Utilizing this theorem of Noether again 10 years later in 1929, Weyl derived Maxwell's equations of electromagnetism from the requirement that the Lagrangian field density remain invariant when the rigid, globally defined U(1) phase group of transformations of the wave function of the electron (invariance of which is required for electric charge conservation) becomes a local phase group, i.e., is 'gauged', becoming a field function depending on the space and time coordinates. Thus was born the modern principle of gauge invariance, the first and canonical illustration that 'local symmetry dictates the form of the interaction'. Then again, there are Weyl's signature contributions to pure mathematics (in particular, on representations of semi-simple Lie groups, 1925-6) that confer his rank as one of the premier mathematicians of the 20th century. Philosophers of mathematics will not fail to point to Weyl's predicative foundation for analysis in 1918 as well as his constructivist proposals and his position, similar but distinct from Brouwer's, during the 1920s debates in the foundations of mathematics.

Raum-Zeit-Materie, Weyl's epochal work on general relativity that went through five German editions in six years (1918-1923), introduced the concept of linear (affine) connection to describe the gravitational-inertial field ('guiding field', in Weyl's anschaulich terminology). As generalized by Élie Cartan, the notion of a connection would become (through the work of Cartan's students C. Ehresmann and S.S. Chern ca. 1950) the core constituent of the modern formulation of the differential geometry and topology of fiber bundles, as well as (ca. 1975) the basis of the geometric formulation of the Yang-Mills gauge theories comprising the Standard Model. Alas Weyl's book exists only in an execrable English translation made in 1922. Readers of the fifth German edition, never translated, know that in an appendix Weyl presented, on astrophysical grounds, an expanding universe model at the same time and independently of A.A. Friedmann, deriving a 'world radius' corresponding to about 1/5 of the value of the constant discovered by Hubble six years later.

With quantum theory, Weyl was also in at the beginning. As Paul Forman recognized in his controversial history of the origins of quantum mechanics (1971), Weyl, though the author of a unified field theory already in 1918 (antedating Einstein's initial efforts), was among the first to realize that the old quantum theory of Sommerfeld, Einstein, Planck and Bohr would have to be scrapped on account of an irreducible element of probability in atomic theory. Anecdotally we know that, developing the linear differential equation that bears his name in the winter of 1925-6, Schrödinger sought Weyl's assistance in deriving the core case of radial eigenfunctions (specifying the distance between electron and nucleus). In 1930 Weyl, commenting on Dirac's relativistic theory of the electron, corrected the latter's assumption that the positively charged particle created by the 'hole in the negative energy sea' was not, in fact, the much more massive proton but an unknown particle having the mass of the electron. Weyl's Gruppentheorie und Quantenmechanik (1928[1], 1931[2]) inaugurated the study, appropriately termed 'Weyl's Program' by the late George Mackey (1988), of quantum structures through the theory of linear representations of discrete and continuous groups. Those who preserve through the second edition's fifth chapter know of Weyl's character formula, his "bridge" (reciprocity relations, expressed by Young tableau) between characters of the symmetric group (which is finite) and the unitary group (which is continuous).

The catalogue of Weyl's scientific achievements might be extended further but it already suffices to show that such a rich harvest of results in the diverse fields of pure mathematics and physics has few equals in the history of science. A question one may at least entertain is whether Weyl's rather intricate philosophical proclivities underlie this extraordinary fruitfulness. But in all honesty, how many contemporary philosophers of science have even heard of, let alone studied, Weyl's Philosophy of Mathematics and Natural Science? First published in German for a handbook of philosophy in 1926 when Weyl was at the peak of his creativity, a revised edition appeared in 1949 rendered into serviceable English by Olaf Helmer, and augmented with five new appendices written by Weyl in English. Out of print for decades, in 2009 Princeton University Press revived this classic of philosophy of science (2009a), with a new introduction by Nobel physicist Frank Wilczek, re-publishing it together with a companion volume (2009b) of Weyl's philosophical essays. In the author's opinion, these are, together with Weyl's little book Symmetry (1952), among the most profound works of philosophy of science produced in the 20th century.

Elsewhere Weyl tells us that he drafted the original edition of (2009a) in 1926 "in a few weeks of vacation", after spending the previous year "browsing in the literature of philosophy … like a butterfly flying from flower to flower, endeavoring to get a bit of honey from each". Like Leibniz, to whom frequent reference is made, Weyl was a synthesizer, seeking to harness, then to harmonize, antagonistic philosophical viewpoints, viz., realism and idealism, the latter in both its positivist and non-positivist forms. The principal task of the book is to show that like mathematics, the natural sciences also have a "constructive character" (2009a, 151), and to provide philosophical elucidation of what this means. Of course, Weyl's constructivist assessment of mathematics and natural (really, physical) science has a transcendental flavor, and indeed Kant is credited "for elevating into philosophical consciousness the conception of reality which dominated the sciences since Galileo", namely, "the attempt to ascertain by a systematic procedure the aprioristic principles for the construction of empirical reality". (2009a, 164) Yet Weyl is not really a pure-hearted disciple, for he finds that the practice of natural science reveals no "clear-cut division into *a priori* and a posteriori in the Kantian sense" but rather "a rich scale of gradations of stability". (2009a, 153-4) Observing that "the natural scientist will find it difficult to be satisfied with (Kant's) attempt", Weyl chose to subordinate Kant to Leibniz, judging that "Kant's transcendental idealism reestablished the insights already gained by Leibniz." (2009a, 122)

The rapprochement of natural science and philosophy is the guiding theme throughout Weyl's philosophical endeavors. Philosophy of Mathematics and Natural Science is, however, a difficult book to read; in a glowing review the logician Hao Wang correctly observed "perhaps few readers could hope to comprehend all the details in this amazing book". (Wang 1949, 35) But, and this makes Weyl particularly relevant to Dummett's challenge, Weyl also reached out to a wider audience, as shown in particular in the essays collected in the companion volume mentioned above, Mind and Nature: Selected Writings on Philosophy, Mathematics, and Physics, and in his little book Symmetry, based on the Louis Clark Vanuxem Lectures given in February 1951 at Princeton on the eve of his retirement from The Institute of Advanced Study. The latter, art historical and biological illustrations aside, is essentially a vulgarisation of the underlying philosophy of Weyl (1938), a classic text of group theory.

In large measure, the essays contained in Mind and Nature, dating from 1921 to 1954, the year before Weyl's death, sound out the philosophical themes he regarded as salient of the epochal changes in mathematics and physics during the four decades of his productive scientific career from 1910-1950. Appropriately, the heart of the book reprints two sets of lectures Weyl delivered in English to lay audiences in the early 1930s: the Terry Lectures at Yale in 1931, subtitled "Three Lectures on the Metaphysical Implications of Science", and the William J. Cooper Foundation Lectures, given at Swarthmore in the autumn of 1933. Each lecture series rehearses (sometimes quoting verbatim but also elucidating and expanding upon) the philosophy of science set out in Philosophy of Mathematics and Natural Science, at that time available only in the original German. The timing of the Swarthmore lectures is significant: Weyl, his Jewish wife Helene Joseph and their two sons, had only just arrived in Princeton, having fled Göttingen in Hitler's Germany. At the Institute of Advanced Study, recently established at Princeton by Abraham Flexner, Weyl joined Einstein and John von Neumann as one of the six founding professors of mathematics. Yet in the early 1930s Weyl was but little known in America beyond a small contingent of mathematicians and theoretical physicists. To be sure, the fourth (1921) edition of Raum-Zeit-Materie had been (ineptly) translated into English in 1922. And while the second (1931) German edition of his book on group theory and quantum mechanics had been (accurately) translated that same year by H.P. Robertson, later to become a leading relativistic cosmologist at Cal Tech, it was largely unread even by quantum physicists, being principally known for inflicting what Pauli termed "die Gruppenpest" on quantum mechanics. Highly respected in Europe as Hilbert's favorite student and his chosen successor, Weyl's name was familiar in America only to those few cognoscenti who followed developments in mathematics and theoretical physics at the highest level. It appears fair to say he was not known at all to period American philosophers. Sadly, that ignorance persists in large measure today.

3. Bridging the Gulfs

The 1931 Yale lectures open with a line from a 1913 poem of Franz Werfel, Eine alte Frau geht (An old woman passes):

> "Diese Welt ist nicht die Welt allein"
> (This world is not the only world.)

Weyl's famed literary erudition here compressed his unifying idea into a single, if cryptic, sentence. His intent is more prosaically expressed at the beginning of the 1933 lectures, when stating their "foremost theme":

> The structure of our scientific cognition of the world is decisively determined by the fact that this world does not exist for itself, but is merely encountered by us as an object in the correlative variance of subject and object. The world exists only as that met with by an ego, as one appearing to a consciousness; consciousness in this function does not belong to the world, but stands out against being as the sphere of vision, of meaning, of image, or however else one may call it. (Weyl 2009b, 83)

An earlier lecture, delivered in German at an International Congress of Philosophy at Harvard in September, 1926, offers a variation on this theme; once again Weyl's emphasis is on the necessary role of human consciousness in constructing what Wilfred Sellars aptly and influentially called 'the scientific image', the theoretical picture of nature created within natural science:

> The immediately experienced is subjective and absolute. On the other hand, the objective world is necessarily relative and may be represented by something definite, numbers or other symbols, only after a coordinate system has been arbitrarily imposed on the world. .... The necessity of the coordinate system goes back to the ultimate

> epistemological fact, the interpenetration of the This (here-now) and the That. This interpenetration is the general form of consciousness: only insofar as continuous extension and continuous quality coincide does something exist. This double nature of that which is real has the consequence that we can only draw up a theoretical picture of that which exists against the background of the Possible. (Weyl 2009b, 31)

In these declarations are found the core principles of Weyl's epistemology and metaphysics of science: neither realist, nor naturalist, nor pragmatist, nor positivist, Weyl drew heavily from transcendental, phenomenological and even existential currents of German idealism, tendencies owing much to Leibniz and Husserl, less to Dilthey, Kant and Fichte, but also even something to Heidegger and to that peculiarly German institution, Lebensphilosophie. Indeed, the Yale lectures set Weyl's epistemological views in a metaphysical setting informed by existential grappling with God. With Hitler's takeover of Germany on the horizon, these lectures conclude with a dark recognition that Heidegger's Dasein – the finite, essentially temporal, being that is "thrown into the world" – has opened a rupture, a yawning abyss, between immanent consciousness, the cognizing and sense-giving "I" of all creative activity, and "the concrete man that I am, who was born of a mother and who will die". To Weyl, this gap can be closed, if at all, only through God; characteristically, Weyl's God is the deity of a constructive mathematician, "the completed infinite [that] cannot and will not be comprehended" by the mind of man.

4. Constructive Cognition

The recurrent subject of both sets of lectures as well as later essays (dating from 1949 and 1954) is Weyl's distinctive conception of the methodology of mathematics and theoretical physics that he alternately referred to as "symbolic construction" (when designating the aim or goal) or "constructive cognition" (designating the process). What might these terms mean? "All knowledge", Weyl affirmed, "while it starts with intuitive description, tends toward symbolic construction". (2009a, 75) In particular, exact natural science, "the most distinctive feature of our culture in relation to other cultures" (2009a, 216), is a symbolic construction, in which "theoretical construction … supplements the given in the interest of totality, and we are no longer forced to use sense data as our building material." (2009a, 122). The canonical example of a symbolic construction is the mathematical continuum, the completed infinite to which our "urges towards totality" compel us, but which can only be represented in symbols. Thus it is real analysis that initially teaches we must "renounce the mystical error of expecting the transcendent ever to fall within the lighted circle of our intuition." (2009a, 66).

However, the full measure of symbolic construction appears first with the "problem of space", where "mathematics, natural science and philosophy permeate one another so intimately". (2009a, 67). Here we learn that:

> A thing exists only in the indissoluble unity of intuition and sensation, through the superimposition of continuous extension and continuous quality. (2009a, 130-1)

Reference to this "dual nature of reality" recurs later on in PMNS as Hilbert's 'Hier-So' ("here-thus") relations, "the description of the world according to field theory", the 'here' given by space-time coordinates, the 'thus' by physical quantities. (2009a, 179) Weyl's analysis of physical objects into an overlap of quality and extension reaches back to Aristotle's tode ti, through Husserl's 'Dies-Da' and on to the 'this-something' fiber bundle structure of Sunny Auyang's ontology for quantum field theory. (Auyang 1995, §19) But Weyl digs deeper; following Husserl, he recognizes the "penetration of the This and the That" as "the general form of consciousness", that "phenomenologically, it is impossible to go beyond".

Extension comes first; symbolic construction begins with construction of the space-time continuum:

> We cannot design a theoretical image of being except upon the background of the possible. Thus the four-dimensional continuum of space and time is the field of the *a priori* existing possibilities of coincidences. (2009a, 131)

In accordance with Weyl's grounding of meaning in intuitive description, "an intuitively evident meaning", on which all symbolic/theoretical construction evidentially rests, can only be assigned "to spatio-temporal coincidence and immediate spatio-temporal proximity." (2009a, 95) Of course construction within a otherwise featureless continuum presupposes that its points be distinguishable by marks or labels, possible only in relation to a coordinate system, or frame of reference. However, a coordinate system is an arbitrary representational contrivance introduced by "an individual demonstrative act". Its telltale trace of subjectivity is a constant reminder that:

> The objectification sought in theoretical construction, the elimination of the ego and its immediate life of intuition, does not fully succeed, and the coordinate system remains as the necessary residue of ego-extinction. (2009a, 75)

Accordingly, constructive cognition insists that objects of cognition in natural science (comprising the objective world) are never simply apprehended or registered by the mind but rather constituted by the mind in theoretical constructions, an obvious concession to idealism:

> Science concedes to idealism that its objective reality is not given but to be constructed (nicht gegeben, sondern aufgegeben), and that it cannot be constructed absolutely but only in relation to an arbitrarily assumed coordinate system and in mere symbols. (2009a, 117)

Weyl's language here is telling, for the distinction nicht gegeben, sondern aufgegeben is readily recognized as that of Kant in the Transcendental Dialectic (A498/B526), where emphasis is placed not on constitutive synthetic *a priori* judgments that are the concern of the Transcendental Analytic but on the regulative use of principles or ideas of pure reason.

Let us recall that the first and by far major portion of The Critique of Pure Reason, "The Transcendental Doctrine of Elements", is divided into two parts, a (relatively much smaller) Transcendental Aesthetic and a (much larger) Transcendental Logic, where the latter is partitioned into two subdivisions: Transcendental Analytic and Transcendental Dialectic. Undoubtedly, the Transcendental Aesthetic and the Transcendental Analytic are the best known parts of the Critique, for the former contains the doctrine that space and time are *a priori* forms of sensibility, while the latter presents Kant's answer to the question of how synthetic *a priori* judgments are possible: namely, as judgments structured by the categories that, as schematized by the forms of intuition, prescribe precise boundaries within which all human cognition of objects occurs. Of course, it is the doctrine of space and time in the Transcendental Aesthetic that most egregiously requires modification in the light of relativity theory.

In arguing that synthetic *a priori* judgments can only be established within the domain of sensible experience, the Transcendental Analytic already initiated a critique of traditional metaphysics, whose *a priori* judgments transcend the boundaries of possible experience. Kant's aim in the Dialectic is somewhat subtler. No mere skeptic of metaphysics, he wished to show that, although the questions that preoccupy metaphysical inquiry are inevitable, as inherent in the nature of human reason itself, they are nonetheless deceptive, and always must be understood in the right (i.e., "critical") manner on pain of falling into metaphysical dogmatism. It is a delicate high wire act, and many have concluded that in the Transcendental Dialectic, Kant did not keep his balance, plunging into confusions that undermine or fatally threaten the carefully constructed argument for transcendental idealism in the Analytic. Though the matter cannot be argued here, the work of the late Gerd Buchdahl (1970) and others has persuaded some of us that the Dialectic is indispensable to transcendental idealism's understanding of post-Newtonian physics, as well as to the Critique as a whole.

Now in the Dialectic, emphasis is not on the constitutive rules of the understanding but on regulative principles, transcendental ideas or concepts of the faculty of reason. (A302/B359) Its most celebrated chapter, on the Antinomy of Pure Reason, shows that the antinomies stem from an uncritically accepted directive of reason to find, for the cognitions provided by the understanding (whose *a priori* conditions are inventoried in the Transcendental Analytic), the unconditioned totality or unity of all such cognitions. However, such an unconditioned totality can, according to the Analytic, never be given as an object of possible experience, in contrast to the assumptions of a dogmatic transcendental realism that treats appearances as things-in-themselves. The resolution of the Antinomy comes through critical reflection upon the demand of reason to bring all cognition under a principle of systematic unity; while the idea of such unity is necessary to the guidance, and so, proper functioning, of the understanding, it is nonetheless not constitutive of an actual object of experience. Rather it is an indispensable regulative idea, a focus imaginarius towards which all cognition is directed, providing an ideal goal and direction of inquiry that mark out the way knowledge is to be sought and organized. The assumption that nature embodies such a unity cannot be disconfirmed by recalcitrant experience, for it is a presupposition of seeking physical laws at all.

Thus the guiding conception of the Transcendental Dialectic is the notion of a "concept of reason", or, what Kant termed "an idea":

> These concepts of reason [i.e., ideas] are not derived from nature; on the contrary, we interrogate nature in accordance with these ideas, and consider our knowledge as defective so long as it is not adequate to them. (A 646-7/ B 673-4)

While such a concept serves as a schema through which other objects (of empirical cognition) are represented indirectly in their systematic unity through their relation to the concept, as a schema it does not have its own actual object (a never-to-be-completed ideal) and is therefore not constitutive of an object of possible experience. From the standpoint of epistemology of science, Kant's aim in invoking such a regulative use of concepts is to demonstrate that empirical knowledge presupposes a general framework of unity within which specific empirical claims can be situated. The regulative use of reason, by specifying the ideal structure of a completed system of scientific knowledge, provides the context within which specific scientific theories are located. In this way, scientific theorizing requires a transcendental, not merely a logical (methodological, instrumental) use of ideas, articulating an ideal explanatory system to which any current knowledge of the world only approximates.

In sum, the regulative use of reason involves a fundamentally different use (and meaning) of *a priori* knowledge than that attributed to the understanding in the Transcendental Analytic. Empirical science requires the presupposition that nature accord with reason's interest in unity; nonetheless, the way, or degree to which, this demand may be satisfied cannot be specified *a priori*. By paying attention to this largely neglected aspect of Kant's account of the nature of empirical knowledge, one comes to see that, despite championing necessary *a priori* judgments in the Analytic, Kant was highly sensitive to the manner in which empirical knowledge is an on-going self-correcting enterprise in which experience plays a central, but by no means the only, role and that regulative ideas of unity, transcending experience, have an independent and essential role to play in constructing physical theories. This is precisely Weyl's understanding of the role of symmetry principles in theoretical physics (cf. 2009a, 159; 1952).

Against this contextual background of the Antinomy of Pure Reason we may now begin to decode Weyl's statement that in science objective reality is not given but only to be constructed (aufgegeben) in "mere symbols" via the arbitrary introduction of a coordinate system. From the conditioned cognitions provided within the understanding, antinomies arise with an uncritically accepted directive of reason to find the unconditioned totality or unity of all such cognitions (corresponding to the realist's sense of objective reality as given). Since according to the Analytic, such an unconditioned totality can never be given (gegeben) as an object of possible experience, the Dialectic argues that it can only be set as a task for construction (sondern aufgegeben). Among period philosophers in the 1920s, Weyl was not unusual in invoking the nicht gegeben sondern aufgegeben contrast, as several (otherwise differing) schools of neo-Kantianism commonly found in it the expression of the very kernel of transcendental idealism. For example, Ernst Cassirer, the most significant representative of the "Marburg" School in the period between the world wars, viewed this distinction as "the transcendental insight":

> The transcendental insight … that the 'absolute' is not so much 'given' as 'posed as a problem'.[5]

This epitome of transcendental idealism is found also in Heinrich Rickert's widely-read book (1921) subtitled an "Introduction to Transcendental Idealism". Rickert is mostly remembered today as the leading figure of the so-called "Southwest" or Baden School of neo-Kantianism and the director of Heidegger's dissertation. According to Rickert,

> For the transcendental idealist, the object of knowledge is …neither immanent nor transcendently 'given', but rather 'posed as a problem'.[6]

To cite yet a further example, the same view recurs in an otherwise forgettable neo-Kantian treatment of relativity theory except for the fact that 1) it was probably read by Einstein, since he reviewed it in 1924; and 2) it forcefully repudiates the usual allegation that transcendental philosophy is wedded to the necessity of Euclidean geometry as the geometry of physical space.

> For if the essential in the Kantian view is that the world is not given, but posed as a problem (nicht gegeben sondern aufgegeben), to be constructed out of the given 'material of sensation' according to principles essential to thinking, and that we have only presupposed as *a priori* that which is necessary to this construction, then it follows from this most general, highest principle of transcendental philosophy that one can safely give up the *a priori* validity of the Euclidean axioms as soon as it is indicated that they do not belong among these essential presuppositions. (Winternitz 1923, 201-2)

It is perhaps worth mentioning that as late as 1949 Einstein himself invoked the contrast nicht gegeben sondern aufgegeben in identifying what is "truly valuable" in

5 "die transzendental Einsicht …daß das 'Absolute' nicht sowohl 'gegeben', als vielmehr 'aufgegeben' ist." (Cassirer 1918, 320)

6 "Der Gegenstand der Erkenntnis ist demnach für dem transzendentalen Idealisten weder immanent noch transzendent 'gegeben', sondern 'aufgegeben'." (Rickert 1921, 316)

Kant.[7]

Where does this leave Weyl's task of constituting objectivity via symbolic construction? Obviously acknowledging that "our knowledge stands under the norm of objectivity" (2009a, 71), constructive cognition, as just seen, denies the full-blown objectification of realism that as Debs and Redhead recently put it (2007, 71) "intuitively involves a sense of subject-independence".[8] In examples from relativity theory and quantum physics, Weyl's constructive constitution of objectivity is repeatedly characterized as requiring the inseparable dualism of subject and object, affirming that the absolute (or objective) is not attainable without the relative (or subjective). (2009a, §13) terms this "the relativity problem"; it is also the underlying theme of Weyl's book Symmetry (1952). But the most concise formulation is given in Weyl (1938,16; original italics):

> The relativity problem: to fix objectively a class of equivalent coordinatizations and to ascertain the group of transformations **S** mediating between them.

The relativity problem has two phases: 1) objectivity is defined as invariance with respect to the group of automorphisms acting on the considered space (pertaining to the abstract group G); and 2) the determination of invariant relations, requiring construction via arbitrary introduction of a coordinate system or set of labels, "self-created, distinctive, and always reproducible symbols" (pertaining to linear representations of G). Thus objectivity requires the subjective: the introduction of a coordinate system is both relative and necessary, as indicated in the quotation above.

Weyl saw this fundamentally novel conception of scientific objectivity stemming from two sources, one in pure mathematics, in transiting from the intuitive continuum (which is featureless, subjective and absolute) to the mathematical continuum (objective and relative), the other in theoretical physics, in transiting from coordinate-free representations or abstract groups to measurable physical quantities and irreducible representations (corresponding to Hermitian operators) of these groups, as in the theory of general relativity and in the application of group theory to quantum mechanics. In the latter, Weyl's relativity problem can now be stated as the *a priori* constructive specification of possible physical states by determining the linearly independent quantities that transform invariantly in the respective irreducible subspaces of Hilbert space.[9] Thus objectivity (the covariant quantities characterizing the state of an object) is not given per se but is constructed by projecting the actual upon an *a priori* - delimited background of possibilities. In so many words, an *a priori* statement in physics lies in formulating in full generality the notion of the possible covariant quantities of a

7 "I did not grow up in the Kantian tradition, but came to understand quite late the truly valuable which is to be found in his doctrine, alongside of errors that today are quite obvious. It is contained in the sentence: "The real is not given to us, but rather put to us (nicht gegeben sondern aufgegeben) (by way of a riddle)." This obviously means: There is such a thing as a conceptual construction for the grasping of the interpersonal, the authority of which lies purely in its validation. This conceptual construction refers precisely to the 'real' (by definition), and every further question concerning the 'nature of the real' appears empty." (Einstein 1949, 680)

8 Debs and Redhead (2007, 61) unfortunately mistake Weyl's views, affirming e.g., "for Weyl the symmetries of nature … including spatial translations, exist as entirely independent from human subjects".

9 In Mackey's paraphrase (1988, 11): "How does one arrive at the self adjoint operators which correspond to various concrete physical observables?"

definite type in the respective constructed manifold or vector space. "Constructive cognition" thus implies constraints on what can be meant by objectivity in physics: as a matter of reflective epistemological judgment, the only permissible meaning that can given to the objective world that science attempts to describe is not the subject-independent world of realism, but of the world of quantities that transform invariantly as portrayed within our best confirmed scientific theories, a world that is de facto a symbolic construction.

5. Conclusion

As indicated above, discourse on the "constitution of objectivity" has been out of fashion, at least in mainstream Anglo-American philosophy of science, since Carnap's Aufbau. A principal result, and lasting legacy, of the logical empiricist treatment of mathematical natural science is the complete dismissal of the problem of the constitution of the object of knowledge in physical theory. Many philosophers today, particularly realists of the Australian variety, concur with Schlick that the transcendental problem, of positing structures within experience that are not found there but which are the results of the active contributions of the mind to experience, simply conflate the conceptual wrapping of reality with reality itself. Such criticism, of course, presupposes we can have some grasp of the notion "reality itself" not in turn reliant on conceptual or discursive structures. But though contemporary scientific realism has faced significant criticism, it remains a dogmatic assumption in much of philosophy of science, to the detriment, in my opinion, of philosophy's engagement with both science itself and with the wider intellectual culture.

The basis for a systematic articulation of an alternative to this current state of affairs in philosophy of science is to be found in the writings of Hermann Weyl, mathematician and interloper in both theoretical physics and philosophy. Swimming against the positivist philosophical climate of the 1930s and, since the 1970s, the default realism of philosophy of physics, for years the only flickers of Weyl's epistemological vision were to be found either within constructive mathematics (in particular, in the work of Solomon Feferman) or within marginal trends of 'continental philosophy' that somehow resisted the fashionable blandishments of Heidegger and his French epigoni. As a result, for entirely contingent reasons, scarcely a trace of his philosophical engagement with science and the educated public has remained perceptible either within physics or philosophy, a cultural amnesia that has greatly contributed to the current impasse over scientific realism. Readers of Weyl will not find a completely worked out philosophy of 20th century physics and mathematics. Such was neither Weyl's aim nor, more importantly, his temperament. Summarizing his last published essay in 1954, he remarked, "An epistemological conscience, sharpened by work in the exact sciences, does not make it easy for the likes of us to find the courage for philosophical utterance." (2009b, 220) Still philosophers who seek to reaffirm the European tradition in philosophy as a shining component of European culture must remain ever grateful that Hermann Weyl, compromising his conscience to the extent that he did, left behind his unrivaled treasure of insights into the murkiest philosophical depths of mathematics and theoretical physics. It is my hope that study of Weyl's profound philosophical engagement with science will assist contemporary thinkers who aspire to see philosophy once more among the jewels of European civilization.

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