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IF MONTY HALL FALLS OR CRAWLS

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ABSTRACT

The Monty Hall problem is consistently misunderstood. Mathematician Jeffrey Rosenthal argues in "Monty Hall, Monty Fall, Monty Crawl" and Struck By Lightning that a proportionality principle can solve and explain the Monty Hall problem and its variants like Monty Fall and Monty Crawl better than the classic solution. Rosenthal's Monty Fall example and solution are examined in detail. I show he has misidentified the crucial assumption in the Monty Hall problem, and his own Monty Fall problem is logically equivalent to the original Monty Hall problem. I then present the Monty Fall* case where the probabilities for which door to pick post tease reveal are actually 50/50 using nothing more than Bayes' Theorem and the standard rules of probability to prove the results—no proportionality principle is needed. The classic solution prevails as explanatorily more powerful. Finally, I show that Monty Crawl is also better explained and solved with the classic solution rather than with Rosenthal's proportionality principle.

Keywords: Monty Hall, Monty Fall, Monty Crawl, Bayes' Theorem, Proportionality Principle, Jeffrey Rosenthal

1. Introduction

The Monty Hall Problem is infamous: Three doors, one prize (originally a car), an initial choice of a door by the contestant, a tease reveal by Monty, and a final decision to switch doors or stay with the original door by the contestant. The correct solution to the classic Monty Hall problem is that the contestant should switch doors after the tease reveal because she will win 2/3 of the time when she switches.[1] The explanation is that the contestant's initial choice had a 1/3 chance of being right, and so the odds she was wrong, given the general negation rule of probability is: 1 minus 1/3 or 2/3. So, switching after Monty reveals an incorrect door is the best thing to do because the contestant wins 2/3 of the time she switches to the remaining unopened door. I shall call this the "classic" solution and claim it is explanatorily superior to Rosenthal's proposed proportionality explanations.

Jeffrey Rosenthal has presented a new version of the problem he calls "Monty Fall" where he claims the "classic" solution, what he calls the "shaky" solution, doesn't really explain the essence of the solution to the Monty Hall problem and close variants. He claims there is a better solution for explaining examples like original Monty Hall as well as the variant Monty Fall:

1 Given the game is fair, Monty knows where the prize is, and Monty doesn't only allow the possibility of switching when the contestant picks the prize door on the initial choice.

> **Monty Fall Problem:** In this variant, once you have selected one of the three doors, the host slips on a banana peel and *accidentally* pushes open another door, which just *happens* not to contain the car. *Now* what are the probabilities that you will win the car if you stick with your original selection, versus if you switch to the remaining door? (Rosenthal 2008, 5)

Rosenthal claims the odds for his Monty Fall post reveal choice are 50/50—it doesn't matter if you switch or stay. He appeals to a principle he names "proportionality."

> **The Proportionality Principle:** If various alternatives are equally likely, and then some event is observed, the updated probabilities for the alternatives are proportional to the probabilities that the observed event *would* have occurred under those alternatives.[2] (Rosenthal 2008, 6)

Rosenthal claims using the proportionality principle produces better solutions than the classic/shaky solution, and can explain Monty Fall and other variants of the problem like Monty Crawl and perhaps Monty Small.[3] I shall show Rosenthal has made a conceptual error in constructing his Monty Fall variant, that Monty Hall and Monty Fall are logically equivalent, and as such must have the same probability, which means the contestant should switch doors to increase her odds of winning to 2/3 in the Monty Fall case too. In doing so, I will not appeal to the proportionality principle, making it irrelevant as part of the explanation and solution to both Monty Hall and Monty Fall; both problems can be easily solved and, more important, explained with Bayes' Theorem and the standard rules of probability. That is, if the proportionality principle says the Monty Fall odds are 50/50, then either it is false or Rosenthal has constructed his Monty variants in such a way as to get the wrong results. Since the proportionality principle is an obscure version of Bayes' Theorem, I argue the latter in Rosenthal's Monty variants.

2. Rosenthal's Two-Move Strategy

In both *Struck By Lightening*[4] and "Monty Hall, Monty Fall, Monty Crawl" Rosenthal used several Monty Hall style cases to help demonstrate and explain the proportionality principle and then claims the classic/shaky solution has an unjustified assumption, which is relevant to the Monty Fall problem. He then claims the proportionality solutions are better. In neither case does he actually provide a calculation, but instead lets the reader intuit or calculate the result themselves.[5] Rosenthal's mistakes are made in the analogies and his additional assumptions, built into his Monty Fall and Monty Crawl problems. Let's look

[2] There is a slightly differently worded version of the proportionality principle in Rosenthal 2006, 210.

[3] Monty Crawl is a variant where after the contestant picks a door, Monty tease reveals by crawling to the closest available door to him that has the lowest number. This problem has an additional assumption in it that makes it not relevant to the current cases, but it can still be explained by the classic solution. I will show this below. Monty Small is sufficiently different from the other cases that considering it is unnecessary for a unified solution.

[4] Especially pages 206 to 215.

[5] Rosenthal does, however, provide one proof in an appendix of "Monty Hall, Monty Fall, Monty Crawl" showing the proportionality principle is an instance of Bayes' Theorem.

at his two important analogies.

> ***Sister in the Shower:*** Either Alice or Betty is equally likely to be in the shower. Then you hear the showerer singing. You know that Alice always sings in the shower, while Betty only sings 1/4 of the time. What is the probability that Alice is in the shower? (Rosenthal 2008, 3)

Rosenthal writes: "By Proportionality Principle…4/5 Alice is in the shower." (Rosenthal 2008, 6)

I take it his proportionality principle works in this case and Alice is likely to be singing in the shower, but there is a relevant disanalogy in this case. One gets new information: singing. There is no new information in the original Monty Hall case. When Monty opens one of the other two doors, revealing nothing behind it, we already knew, *a priori*, at least one of the two doors didn't have anything behind it. We have no new information, like singing. The sister in the shower case isn't like the Monty Hall case, but this is just Rosenthal's first move to get us comfortable with the proportionality principle. The second case, "Three-Card Thriller," is where Rosenthal makes his big move.

> ***Three-Card Thriller***: A friend has three cards: one red on both sides, one black on both sides, and one red on one side and black on the other. She mixes them up in a bag, draws one at random, and places it on the table with a red side showing. What is the probability that the other side is also red? (Rosenthal 2008, 6)

Here Rosenthal has a neat solution using the proportionality principle. He says that if we take the probability that each of the cards has a red alternate side (red/red, black/black, and red/black), then we get 1, 0 and 1/2. And using the proportionality principle while making the probabilities add up to 1 we get 2/3, 0, and 1/3 respectively. This is the short explanation Rosenthal gives in the paper on how to solve Three-Card Thriller.

In *Struck By Lightning*, Rosenthal gives a fuller account of this case. In this example, he has the card shark Charlie say: "So I guess you didn't get the black/black card, eh? You got either red/red or red/black and it must be 50/50 whether or not the other side is red or black, right?" (Rosenthal 2006, 211) But this is to misconstrue the problem just like people do in the Monty Hall case. What people fail to see in the Three-Card Thriller case is that the red/red card needs to be counted twice in the calculation because each side is red. So, it does not matter if you ask the question about red or black if you are looking at the color in question on your card; it is 2/3 likely the same color is on its opposite face, whether red or black.

But Rosenthal does give an explanation as to why he thinks the classic/shaky solution isn't as good as his proportionality principle solution. He says the classic/shaky solution requires an additional assumption:

> **Rosenthal's Additional Assumption:** If the host has a choice of which door to open (i.e., if your original selection was correct), then he is equally likely to open either non-selected door. (Rosenthal 2008, 6)

This additional assumption plays no role in these cases, but since Rosenthal gets the same solution as the classic/shaky solution in the Monty Hall case, that solution isn't worth examining. What isn't clear, however, is how he generates the 50/50 answer for his Monty Fall case. So, I will quote from him at length.

> In the Monty Fall problem, suppose you select Door #1, and the host then falls against Door #3. The probabilities that Door #3 happens not to contain a car (prize), if the car (prize) is behind Door #1, #2, and #3, are respectively 1, 1, and 0. Hence, the probabilities that the car (prize) is actually behind each of these doors are respectively 1/2, 1/2, and 0. So, your probability of winning is the same whether you stick or switch. (Rosenthal 2008, 6)

This is Rosenthal's explanation/solution to the Monty Fall problem, and he is making the same mistake he claims people make in the Three-Card Thriller case explained in *Struck By Lightning*. I claim Rosenthal's version of Monty Fall is, accidentally, structured as logically equivalent to the regular Monty Hall problems. Rosenthal has misidentified the additional assumption in Monty Hall problems, and he has a second assumption in Monty Fall he doesn't account for when he calculates the probabilities with his proportionality principle.[6] This, I believe, is a result of two things: his misidentification of the additional assumption as the cause of the initial problem, and his complete familiarity with the correct answer to the Monty Hall problem and various other puzzles.

3. Revisiting Original Monty Hall

In the original Monty Hall problem, we know Monty always tease reveals a door that doesn't have the prize behind it. This leaves the contestant with their original picked door and one unopened/unpicked door. The most common mistake is to think in these cases it doesn't matter which door you pick post tease reveal because the odds in picking the correct door are 50/50. But this is to misunderstand the classic Monty Hall problem because it does have an additional assumption; it is:

> **The Bad Door Tease Reveal Assumption:** No matter what door the contestant initially picks, Monty will always tease reveal, a non-selected door that doesn't have a prize behind it.[7]

So, when Marilyn vos Savant gave multiple defenses of her versions of the classic solution,

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6 Rosenthal makes an additional assumption and similar mistake in Monty Crawl.

7 This is to claim there is a zero probability Monty will reveal the prize, i.e., Monty never opens the door with the prize because then the game would be over and there would be no opportunity for the contestant to choose to switch or stay, which would be bad television.

promoting switching, in *Parade Magazine*[8] instances of the Monty Hall problem, she had the bad door tease reveal as a background assumption. It is the essential assumption along with the assumption that Monty is being fair by always allowing people the chance to switch instead of just allowing the choice when people picked the winning door to start.

The way I typically explain the classic solution to undergraduate students and friends is: Imagine instead of Monty revealing a door, he offers you both of the doors you didn't pick. You know *a priori* one of those doors cannot have the prize behind it. So, when Monty reveals one of the unpicked doors that doesn't have a prize behind it, you haven't learned any new information about the set of doors you didn't pick. If you wanted the two doors you didn't choose before Monty tease reveals (and you do), then you should want to switch after the reveal—it's the same deal. The odds of winning are 2/3 if you switch because you only had a 1/3 chance of being right in your initial door choice. Here are two simple tables to show the sample space for the instances where "Door 1" has the prize behind it:

Table 1: *Always Stay Example*

Initial choice	Monty Opens	Final Choice	WIN	Probability
Door 1	Door 2	Door 1	Yes	1/6
Door 1	Door 3	Door 1	Yes	1/6
Door 2	Door 3	Door 2	No	1/3
Door 3	Door 2	Door 3	No	1/3

Total wins: 1/3 of the time

Table 2: *Always Switch Example*

Initial choice	Monty Opens	Final Choice	WIN	Probability
Door 1	Door 2	Door 3	No	1/6
Door 1	Door 3	Door 2	No	1/6
Door 2	Door 3	Door 1	Yes	1/3
Door 3	Door 2	Door 1	Yes	1/3

Total wins: 2/3 of the time

What is interesting to note in these cases is that when the contestant initially picks the correct door, Monty has two doors from which to choose to reveal, but when the contestant picks the wrong door, Monty only has one door he can tease reveal—lest he shows the prize. Rosenthal thinks this is why mistakes are made, and why he believes the classic solution fails as both a solution and an explanation. I don't want to speculate as to why people make mistakes with this problem because unlike good reasoning that is easy to categorize, bad reasoning can be particularly hard to categorize, i.e., never underestimate people's ability to be creative when making mistakes in reasoning—even oneself. The

[8] The confusion started when Marilyn vos Savant gave a simplified version of the classic solution and was rebuked for being wrong by professional mathematicians all over the world even though she was right. A full record of her discussion on the topic is provided in the references.

point is Rosenthal has identified the wrong crucial assumption in these cases. Even though this is an interesting fact about Monty's choices for doors he can tease reveal when the contestant initially picks the prize door, it does no explanatory work in explaining the results or explaining the solution, and this is one reason why Rosenthal's solution is not as good as the classic solution.

4. Rosenthal's Mistakes

Although I won't categorize all the mistakes in reasoning for Monty Hall cases, I will explain what Rosenthal has done in making a mistake with setting up Monty Fall. First, I claim his description of the Monty Fall problem makes it logically equivalent to the Monty Hall problem. Second, I claim when he applies the proportionality principle, he is actually applying it to an instance of a slightly different version of the Monty Fall problem I will call Monty Fall*. I will show Monty Fall* does have a 50/50 outcome after a fall reveal and can be calculated using Bayes' Theorem and the standard rules of probability, but Monty Fall* isn't Monty Fall. And the proportionality principle, as clever as it is mathematically, adds a level of complexity to the explanations of these cases, but fails to be clearer, simpler, and more powerful than the classic solution, making an appeal to the proportionality principle otiose.

4.1 Monty Fall is Monty Hall (accidentally)

Monty Fall is set up like Monty Hall with one new twist: "once you have selected one of the three doors, the host slips on a banana peel and accidentally pushes open ***another*** (my bold emphasis) door, which just *happens* not to contain the car (prize)." (Rosenthal 2008, 5) The fact that Monty's fall never reveals the prize, and he reveals a door different from the original door (that's why it's another door) makes the Monty Fall problem logically equivalent to the original Monty Hall problem. Let me explain.

In cases where the contestant picks the door with the prize behind it in the first place, Monty never reveals it, but reveals ***another*** door: in these cases one wins by staying and loses by switching—1/3 and 2/3 respectively. In the case where the contestant doesn't initially pick the prize door, Monty's fall reveals the only door it can that isn't the prize—win 2/3 of the time by switching. This makes Monty Fall and Monty Hall the same in all instances; they are logically equivalent, and thus must have the same probabilities.

Rosenthal's mistake is that his Monty Fall problem has built into the scenario the bad door tease reveal assumption, which produces the 2/3 should switch results and not the innocuous "additional assumption" Rosenthal proposes as the culprit. Essentially, if there is a zero probability of Monty revealing the prize in his fall or otherwise, you should switch and win 2/3 of the time. This is exactly what Rosenthal has done. If one is really familiar with the Monty Hall case, one might be convinced by a cursory reading of Rosenthal's Monty Fall, but closer reading reveals the background assumption that Rosenthal has failed to identify as doing the work in the examples. He makes this mistake, I believe, because of his mistaken additional assumption and his familiarity with the Monty Hall problem—that is, Rosenthal knows he needs a switch explanation.

4.2 Monty Fall and the 50/50 Answer*

If, however, we remove the bad door tease reveal assumption from the Monty Fall problem, we get, what I will call, the Monty Fall* problem. This version gives us an instance where the post fall reveal odds are 50/50. The following is the case I claim Rosenthal wanted to present and solve with his proportionality principle.

> **Monty Fall*** In this variant, once you have selected one of the three doors, Monty slips on a banana peel and accidentally pushes open one of the doors.

In this case, like the others, you can only switch or stay after a reveal happens. If Monty reveals the prize accidentally through his fall, you can neither switch nor stay. This happens in 1/3 of the cases. Another way to think of this is to say that in Monty Fall*, there is a non-zero probability Monty will reveal the prize, and this changes the problem dramatically. That is, Monty's fall could open any of the three doors and in 1/3 of the cases his fall reveals the prize. This immediately ends the game, but if we remove these prize reveal cases from consideration—they are 1/3 of the cases remember, then yes, the odds after Monty reveals a door through an accidental fall where he opens randomly a door not containing a prize will be 50/50. When Monty falls and opens the contestant's initial choice door and it isn't the prize door, then game doesn't have to end if Monty wants to give the contestant a choice of doors. If Monty does give the contestant a choice of doors after accidentally falling and opening the initially selected door that doesn't contain the prize, then in those cases where Monty allows the game to continue, the odds are now 50/50.

Another way to think of this is to see that in half of the 2/3 cases you should switch in the original Monty Hall version, Monty, in the Fall* cases, reveals the prize, accidentally, thereby ruining your 1/3 switch advantage. 1/2 of 2/3 is 1/3 and if the remaining two doors are equally likely now (1/3 each), then the updated probability really is 50/50. But this English explanation isn't nearly as satisfying as the mathematical proofs; so let's use Bayes' Theorem and the general rules of probability to solve and explain both Monty Hall and Monty Fall*.

5. Solving Monty Hall & Monty Fall* with Bayes' Theorem

I am going to assume the reader understands the law of total probability, the general negation rule, and Bayes' Theorem at this point, and its simplified version as a conditional probability. Let's give the following definitions.

> A: The event that the contestant's initial choice is the prize door.
> B: Monty presents a tease reveal by opening a non-prize door after the contestant's initial choice.

Given this is the original Monty Hall case, we know Monty has a zero probability of revealing the prize—our bad door tease reveal assumption. That is, the probabihlity of B|A is 1. This means the odds of picking the correct door is only 1/3.

So the original odds of picking the correct door is 1/3, and using the general negation rule, we know the odds of not picking the correct door is 2/3, which means one should switch if given the opportunity. This, too, is what is occurring in Rosenthal's Monty Fall. Now in Monty Fall* we need the law of total probability. The law of total probability turns the conditional probability into the full version of Bayes' Theorem, and looks like this:

[9]We know if ¬A means you didn't pick the prize door, but instead picked a door with nothing, then the probability of B is now 2/3. Just multiply 1/3 times 1 added to 2/3 times 1/2 and you get 2/3 using the rule of total probability. So when we remove the bad door tease reveal assumption, the *P*(B) is no longer 1, but is now 2/3. When we recalculate with the Monty Fall* probability using the correct probability for B, we get an answer of 1/2 in the following way:

So Monty's fall as described in Monty Fall* does change the game, dramatically. The important explanatory assumption wasn't what Rosenthal thought it was, the additional assumption, but was instead the bad door tease reveal assumption. The tease reveal assumption is doing the work in both the original Monty Hall problem and the Monty Fall problem, and this is what Rosenthal misses, leading him to look for a better solution and explanation, but there isn't one.

6. The Monty Crawl Variant

In an effort to show what Rosenthal thinks is the superior explanatory power of the proportionality principle, he provides a variant of the Monty Hall problem he names, Monty Crawl. He is particularly sloppy in his presentation of the problem and explaining how the proportionality principle solves Monty Crawl. I show his solution employing the proportionality principle doesn't really explain the Monty Crawl cases. Moreover, his presentation builds in a new assumption that gets him the answer he wants, but it doesn't explain the counterintuitive results. Understanding the classic solution for Monty Hall actually allows for a better understanding of the Monty Crawl variant and the strategies for maximizing one's chance of winning.

> **Monty Crawl Problem:** As in the original [Monty Hall] problem, once you have selected one of the three doors, the host then reveals one non-selected door which does not contain the car. However, the host is very tired, and *crawls* from his position (near Door #1) to the door he is to open. In particular, if he has a choice of doors to open, then he opens the *smallest number* available door. (Rosenthal 2008, 5)

In the Monty Crawl scenario, we have a "new" assumption that changes what we know about the problem in interesting ways, but it doesn't actually change the best strategy for winning, which is still to switch after the tease reveal and win 2/3 of the time, but more

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[9] The law of total probability relies on a logical equivalence: *p* is logically equivalent to (*p* & *q*) v (*p* & ¬*q*), and you can see how this is used to convert the standard conditional probability into Bayes' Theorem quite elegantly in Brian Skyrms' *Choice and Chance* page 131.

on this below. The new assumption in the Crawl variant is the "small door" assumption.

> **Small Door Assumption:** Monty always crawls to and opens the smallest numbered available non-prize door.[10]

For example, if a contestant initially picked Door #1 and it happened to contain the prize, then Monty would always open Door #2. Rosenthal then explains how he believes the proportionality principle solves and explains Monty Crawl. Rosenthal writes:

> The Monty Crawl problem seems very similar to the original Monty Hall problem; the only difference is the host's actions when he has a *choice* of which door to open. However, the answer now is that if you see the host open the higher-numbered unselected door, then your probability of winning in 0% if you stick, and 100% if you switch. On the other hand, if the host opens the lower-numbered unselected door, then your probability of winning is 50% whether you stick or switch. Why these different probabilities? Why does the Shaky Solution not apply in this case? (Rosenthal 2008, 5-6)

Notice that Rosenthal has only described those cases where Monty has a choice of two doors to open. This choice only happens if the contestant initially picks the prize door. If the contestant picks a non-prize door, then Monty doesn't actually have a choice of which door to open, it is determined for him, but I'll come back to this important fact below. Let's look at what Rosenthal writes next as part of his proportionality principle explanation.

> In the Monty Crawl problem, suppose again that you select Door #1. The probabilities that the host would choose to open Door #3, if the car were behind Door #1, #2, and #3, are respectively 0, 1, and 0. Hence, if the host opens Door #3, then it is certain that the car is actually behind Door #2. On the other hand, the probabilities that the host would choose to open Door #2 are respectively 1, 0, 1. Hence, if the host opens Door #2, then probabilities are now 1/2 each that the car is behind Door #1 and Door #3. (Rosenthal 2008, 6)

Rosenthal believes that his proportionality principle solves and explains these problems better than the classic solution. He believes this because Monty has instances where he can choose between two doors in the cases where the contestant initially picks the prize door—the additional assumption. We know that assumption isn't driving the game, but rather the nature of the tease reveal assumption. In an effort to explain both the Monty Hall and Monty Crawl games, it will be useful to break the cases into two types: contestant's

initial door choice is the prize door cases and the contestant's initial door choice is not the

[10] This "new" assumption is really a modification of the tease reveal assumption. Monty can't open the contestant's initial choice door or the prize door. Rosenthal doesn't realize he has actually modified the game by placing Monty near Door #1 and changing the tease reveal assumption because he doesn't realize the tease reveal assumption is driving the original problem. Remember that Rosenthal mistakenly believes the "additional assumption" of door choices when the contestant initially chooses the prize door is what drives the original Monty Hall problem. Rosenthal carries the mistaken "additional assumption" into the Monty Crawl variant.

prize door cases. We know that the probability of the contestant initially picking the prize door is 1/3, and the odds that she does not pick the prize door on the initial choice is 2/3. That accounts for all the possible cases—there are nine in all.

Table 3: *Contestant's Initial Choice is the Prize Door*

#	Initial Choice	Prize Door	Stay Result	Switch Result	Probability
(i)	Door 1	Door 1	Win	Lose	1/9
(ii)	Door 2	Door 2	Win	Lose	1/9
(iii)	Door 3	Door 3	Win	Lose	1/9

When the contestant initially picks the prize door, the contestant always wins if she stays and always loses if she switches. This happens 1/3 of the time because she had a 1/3 chance of being right with her initial choice of doors. But she also had a 2/3 chance of being wrong. So let's look at the cases where she does not initially pick the prize door.

Table 4: *Contestant's Initial Choice is NOT the Prize Door*

#	Initial Choice	Prize Door	Stay Result	Switch Result	Probability
(iv)	Door 1	Door 2	Lose	Win	1/9
(v)	Door 1	Door 3	Lose	Win	1/9
(vi)	Door 2	Door 1	Lose	Win	1/9
(vii)	Door 2	Door 3	Lose	Win	1/9
(viii)	Door 3	Door 1	Lose	Win	1/9
(ix)	Door 3	Door 2	Lose	Win	1/9

Notice that the contestant always wins when she switches, which happens in 2/3 of the cases. That is, in six of the nine possible outcomes that exist, she wins when she switches. This is another way to explain the classic solution to the Monty Hall problem. In 1/3 of the cases she initially picks the prize door and wins when she stays. In the other 2/3 of the cases where she didn't initially pick the prize door, she wins when she switches. So the best strategy to win the prize is to pick a door and then switch to the non-tease reveal door when given the opportunity to switch. The contestant wins 2/3 of the time, that is, those times the contestant didn't initially choose the prize door. The contestant loses 1/3 of the time she switches, and those are the instances where she initially picked the prize door.

So we need to ask: how does the Small Door Assumption change the game in Monty Crawl? In one sense it doesn't change anything about the best strategy to win the prize. If you employ the always switch strategy in Monty Crawl cases, you will win 2/3 of the time. If you employ the always stay method in Monty Crawl cases, you will win 1/3 of the time just as in the Monty Hall cases. This is counterintuitive because it does at times seem like the post tease reveal odds should be 50/50. Remember that Rosenthal writes:

"In particular, if he (Monty) has a choice of doors to open, then he opens the *smallest number* available door." (Rosenthal 2008, 5) What we learn is that Monty only has a choice of doors to open in cases where the contestant initially picks the prize door. These are illustrated in Table 3: Case (i) Monty would open Door #2; case (ii) Door #1; and case (iii) Door #2. In these cases, you learn no additional information about the situation you are in or which door contains the prize. You win if you stay, and you lose if you switch, but this is only 1/3 of the total cases.

What is interesting is that Rosenthal thinks that Monty having a choice of doors, his additional assumption from above, has something to do with the odds of winning when in fact, it is still the tease reveal assumption that is doing all the work. When you include the Small Door Assumption as part of the tease reveal, you learn something in only three of the cases. What is important to realize is that these three cases are all switch-to-win cases. Remember that Monty can't reveal the contestant's initial choice door, and he can't open the prize door. When the contestant's initial choice door and the prize door are not the same and that happens in 2/3 of the cases, then in 2/3 of the cases Monty's tease reveal door is forced upon him. The Small Door Assumption is not a factor in which door he opens as a tease reveal in the 2/3 switch-to-win cases. But what the Small Door Assumption does is allow the contestant to learn when she is definitely in a switch-to-win situation. This new knowledge can only happen in switch-to-win cases; it never happens in stay-to-win cases. And it happens in exactly 1/2 the switch-to-win cases.

In the six switch-to-win cases, (iv)-(ix) from Table 4, the contestant's initial choice is not the prize door. Of these six cases, there are three special cases. In case (iv), Monty must tease reveal Door #3. Why? Well, the contestant initially picked Door #1 and the prize is behind #2. So, the contestant learns that she is definitely in a switch-to-win scenario. If her initial choice of Door #1 was right, then the small door tease reveal would have had to be Door #2. It isn't, so we know she is in a switch-to-win scenario. In (vi), she learns she is in a switch-to-win scenario again. The initial choice is Door #2 and the prize is #1, so Monty must tease reveal #3. We know that the small door assumption should have made Monty open #1 if available, but it isn't, so #1 must be the prize door. And in case (viii) where the contestant initially picks Door #3 and Monty opens #2, we learn the contestant is certainly in a switch-to-win scenario and should switch to Door #1.

So the Small Door Assumption allows us to learn that in the 2/3 switch-to-win cases, we will know we are in a switch-to-win case 1/2 the time. We know that anytime Monty shows Door #3, we are in a switch-to-win scenario. And if we initially choose Door #3 and Monty shows #2, we are in a switch-to-win scenario. What this effectively does is make it clear that if we don't know that we are in a switch scenario, then we will be left with an updated probability of 50/50. But we don't learn this via the proportionality principle. We know it because we know that we only have a 1/3 chance of winning if we stay because our odds of initially picking the prize door was 1/3. If we choose the switch strategy, then we know we will win 2/3 of the time. But if we add the Small Door Assumption, in half of the 2/3 switch-to-win cases, we will know we are in switch-to-win scenario. It doesn't change the 2/3 odds that we will win if we switch, just that we will know, after the tease reveal, half of the switch-to-win cases. What Rosenthal has realized is that if we eliminate from consideration the known switch-to-win cases, then we will win 50% of the time we switch and 50% of the time we stay. But these two 50%s are really just the two 1/3 groups that are left over when we eliminate half of the switch-to-win cases. In the end, in Monty Crawl cases, if you stay, you win 1/3 of the time, and if you switch you win 2/3 of the time. But in half of the 2/3 of the switch-to-win cases, you will absolutely know, with 100% certainty, you are going to win by switching. If you remove those cases, then you are in a scenario where the updated probability is 50/50. Why? Because you can eliminate the 1/3 known switch-to-win cases. That leaves the other 2/3 cases, the stay-to-win cases and the unknown switch-to-win cases, and they are both equally likely making the updated odds 50/50.

Rosenthal asks, "Why does the Shaky Solution not apply in this case?" (Rosenthal 2008, 5-6) The answer is simple: It does. Rosenthal just doesn't realize it. We can see from Table 5 below the different Monty variants and the corresponding probabilities. In the Crawl cases, I have indicated those instances where the contestant will know with 100% certainty that they are in a switch-to-win situation with lowercase 'k' for knowledge. The three instances where there is a 0/6 probability of Monty tease revealing the door, because of the Small Door Assumption, are marked with a lowercase 's' for small. These zero probability doors are different from the other zero probability doors marked as 'not possible', for those so marked are not possible because Monty can't reveal the contestant's initial choice door in any of the variants except accidentally in the Fall* cases.

Let's briefly review and compare the four scenarios presented in Table 5 below. We realize that Monty Hall and Monty Fall have the same probabilities in all cases. We realize that Monty can't open the contestant's initial door choice, and so those odds are essentially shifted to the other two switch cases making the switch-to-win cases each 1/3 likely. The odds of picking the prize door are 1/3, and so the two cases where the contestant stays have to add up to 1/3 as well. So each of those cases are each 1/6 likely. When we look at Monty Fall* we realize that eliminating the condition that Monty can't open the contestant's initial door actually makes the post fall reveal a 50/50 proposition. All of this is easily seen from the work above using the classic solution, but modifying it to allow a non-zero probability that Monty reveals the prize. Neither Rosenthal's additional assumption nor his proportionality principle do any of the explanatory work. Finally, when we get to the Crawl cases, we see that, just like Hall and Fall, Monty can't reveal the contestant's initial door choice. The interesting bit occurs when we realize that the classic solution of switch-to-win works 2/3 of the time. It's just like classic Monty Hall. The difference is that when the contestant initially picks the prize door, Monty never reveals the larger number door. This is essentially a modification of Rosenthal's "additional assumption" that he thinks is driving original Monty Hall. But this doesn't have any affect on the best strategy to win the prize. What it does do, however, is allow for people to learn when they are in a switch-to-win situation with 100% certainty. What we realize, in Monty Crawl cases, is that if you don't know you are in a switch-to-win situation, you have two doors to pick from and the updated odds are 50/50. Rosenthal's belief in the "additional assumption" has made him think he has a better explanation than the classic solution, when he doesn't.

Table 5: *Comparing The Monty Variants*

Initial Choice, Monty Opens, Final	M. Hall	M. Fall	M. Fall*	M. Crawl
Initial 1, Open 1, Pick 2	Not Possible	Not Possible	1/6	Not Possible
Initial 1, Open 1, Pick 3	Not Possible	Not Possible	1/6	Not Possible
Initial I, Open 2, Pick 1	1/6	1/6	1/6	2/6
Initial 1, Open 3, Pick 1	1/6	1/6	1/6	0/6 s
Initial 1, Open 2, Pick 3	1/3	1/3	1/6	1/3
Initial 1, Open 3, Pick 2	1/3	1/3	1/6	1/3 k
Initial 2, Open 2, Pick 1	Not Possible	Not Possible	1/6	Not Possible
Initial 2, Open 2, Pick 3	Not Possible	Not Possible	1/6	Not Possible
Initial 2, Open 1, Pick 2	1/6	1/6	1/6	2/6
Initial 2, Open 3, Pick 2	1/6	1/6	1/6	0/6 s
Initial 2, Open 1, Pick 3	1/3	1/3	1/6	1/3
Initial 2, Open 3, Pick 1	1/3	1/3	1/6	1/3 k
Initial 3, Open 3, Pick 1	Not Possible	Not Possible	1/6	Not Possible
Initial 3, Open 3, Pick 2	Not Possible	Not Possible	1/6	Not Possible
Initial 3, Open 1, Pick 3	1/6	1/6	1/6	2/6
Initial 3, Open 2, Pick 3	1/6	1/6	1/6	0/6 s
Initial 3, Open 1, Pick 2	1/3	1/3	1/6	1/3
Initial 3, Open 2, Pick 1	1/3	1/3	1/6	1/3 k

7. Conclusion

My conclusions are simple. Making mistakes with probability is easy. Clearly showing all of one's assumptions in the Monty Hall case and close variants is difficult. Rosenthal's proportionality principle and additional assumption are interesting but unnecessary to solve and explain the Monty Hall problem. Rosenthal's Monty Fall version isn't actually what he thinks it is, and thus, he mischaracterizes his own problem and produces the wrong explanation for the right answer to the wrong problem. So, in the original cases of both Monty Hall and Monty Fall, we win 2/3 of the time we switch after the tease reveal. In cases where there is a non-zero probability Monty will reveal the prize, like Monty Fall*, the odds do in fact change to 50/50 after the fall reveal, but that wasn't Rosenthal's Monty Fall problem. We also don't need the more complex proportionality principle as part of the explanation because we have Bayes' Theorem and the standard rules of probability to provide a "classic" solution to a not so standard set of problems. Finally, with Monty Crawl, we learn the classic strategy of switching still wins the prize 2/3 of the time and is the best strategy. But in half of the switch-to-win cases we will know, thanks to the small door assumption, that we are in a switch-to-win cases. In the remaining cases where we know that we are not in 100% certain switch-to-win case, the updated odds are 50/50. An interesting fact, but overall, we win when we switch 2/3 of the time in Monty Crawl just like Monty Hall and Monty Fall. The classic solution prevails. It explains the cases, and it is easier to understand than Rosenthal's proportionality principle.

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